



Angles of the CKM unitarity triangle measured at Belle

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Joint Experiment/Theory Seminar, Fermilab
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- intro/some formalism
- measuring ϕ_1 (β)
 $b \rightarrow c\bar{s}s$ tree, $b \rightarrow s\bar{q}q$ penguin
- measuring ϕ_2 (α)
 $B^0 \rightarrow \pi^+\pi^-$, $B^0 \rightarrow \rho^+\pi^-$, $B^0 \rightarrow \rho^+\rho^-$
- measuring ϕ_3 (γ)
 $B^+ \rightarrow D^0 K^+$, $D^0 \rightarrow K^+ K^-$ (GLW), $D^0 \rightarrow K^+ \pi^-$ (ADS)
 $D^0 \rightarrow K_s \pi^+ \pi^-$ (Dalitz plot), $B^0 \rightarrow D^{*+} \pi^-$ ($2\phi_1 + \phi_3$)
- summary: *does the triangle close?*

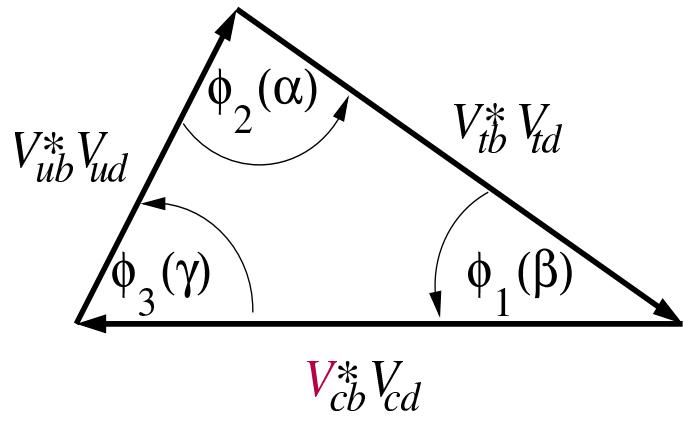
CKM weak mixing matrix:

$$U = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$U^\dagger U = 1 \Rightarrow$$

$$\begin{aligned} V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} &= 0 \\ V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} &= 0 \\ V_{cd}^* V_{td} + V_{cs}^* V_{ts} + V_{cb}^* V_{tb} &= 0 \end{aligned}$$

$$\begin{aligned} V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} &= 0 \\ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} &= 0 \\ V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} &= 0 \end{aligned}$$



$$\begin{aligned} \phi_1 (\beta) &= \arg \left(\frac{V_{cb}^* V_{cd}}{-V_{tb}^* V_{td}} \right) \\ \phi_2 (\alpha) &= \arg \left(\frac{V_{tb}^* V_{td}}{-V_{ub}^* V_{ud}} \right) \\ \phi_3 (\gamma) &= \arg \left(\frac{V_{ub}^* V_{ud}}{-V_{cb}^* V_{cd}} \right) \end{aligned}$$

Does the triangle close? i.e., are there >3 generations?

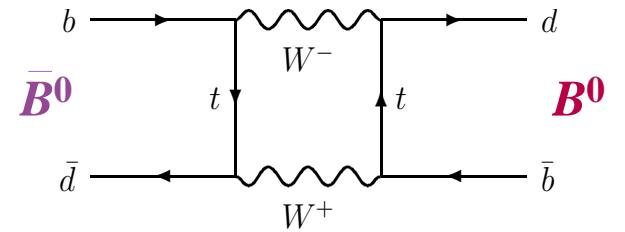


Introduction II

(some formalism)

$$\begin{aligned} |B_H\rangle &= p|B^0\rangle - q|\bar{B}^0\rangle \\ |B_L\rangle &= p|B^0\rangle + q|\bar{B}^0\rangle \end{aligned}$$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - (i/2)\Gamma_{12}^*}{M_{12} - (i/2)\Gamma_{12}}} \approx \sqrt{\frac{M_{12}^*}{M_{12}}} = e^{i2\phi_1} \quad (\text{phase of } V_{td}^* V_{tb})$$



$$\begin{aligned} |B^0(t)\rangle &= e^{-(\Gamma/2+i\bar{m})t} \left[\cos\left(\frac{\Delta m}{2}t\right)|B^0\rangle + \left(\frac{q}{p}\right)i \sin\left(\frac{\Delta m}{2}t\right)|\bar{B}^0\rangle \right] \\ |\bar{B}^0(t)\rangle &= e^{-(\Gamma/2+i\bar{m})t} \left[\left(\frac{p}{q}\right)i \sin\left(\frac{\Delta m}{2}t\right)|B^0\rangle + \cos\left(\frac{\Delta m}{2}t\right)|\bar{B}^0\rangle \right], \end{aligned}$$

$$|\langle f|H|B^0(t)\rangle|^2 = \frac{|\mathcal{A}_f|^2 e^{-\Gamma t}}{2} [1 + |\lambda|^2 + (1 - |\lambda|^2) \cos(\Delta m t) - 2 \operatorname{Im} \lambda \sin(\Delta m t)]$$

$$|\langle f|H|\bar{B}^0(t)\rangle|^2 = \frac{|\mathcal{A}_f|^2 e^{-\Gamma t}}{2} [1 + |\lambda|^2 - (1 - |\lambda|^2) \cos(\Delta m t) + 2 \operatorname{Im} \lambda \sin(\Delta m t)]$$

$$\lambda = \left(\frac{q}{p}\right) \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} = e^{i2\phi_1} e^{i2\phi} \quad (\text{one weak phase})$$



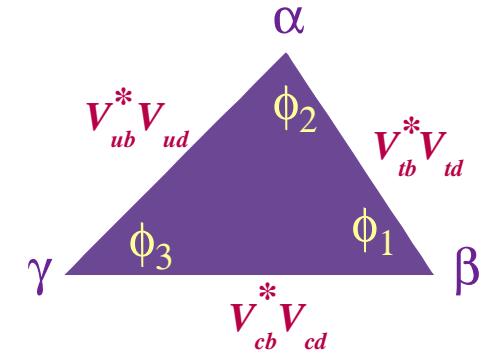
Introduction III

$$\frac{N_{\bar{B}^0 \rightarrow f} - N_{B^0 \rightarrow f}}{N_{\bar{B}^0 \rightarrow f} + N_{B^0 \rightarrow f}} = \mathcal{A}_f \cos(\Delta m \Delta t) + \mathcal{S}_f \sin(\Delta m \Delta t)$$

$$\mathcal{A}_f = \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \quad \mathcal{S}_f = \frac{2 \operatorname{Im} \lambda}{1 + |\lambda|^2}$$

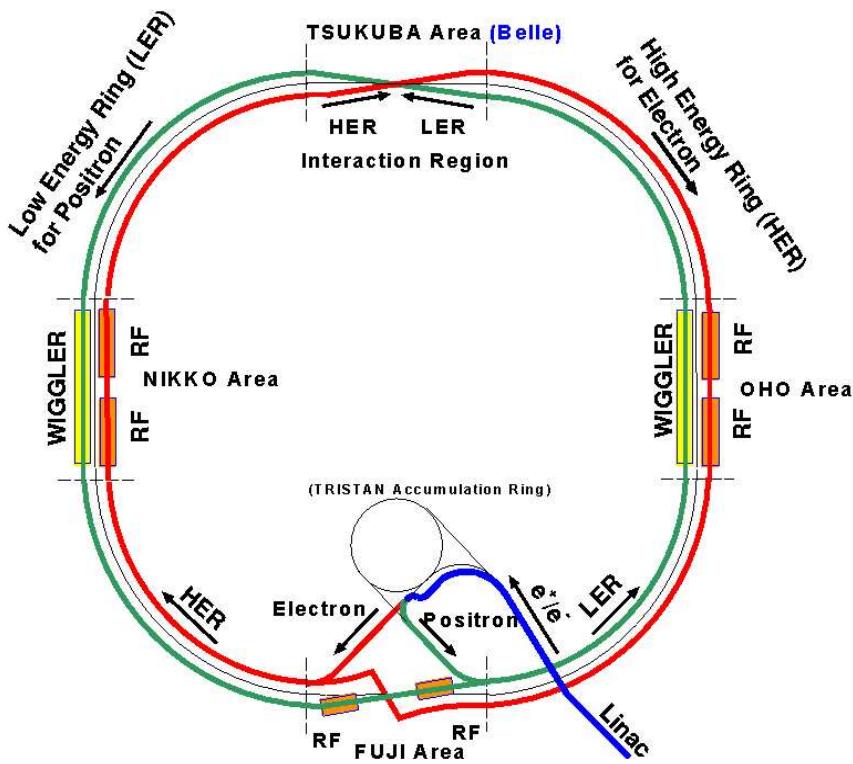
$$\lambda = \left(\frac{q}{p} \right) \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} = e^{i2\phi_1} e^{i2\phi} \quad (\text{one weak phase})$$

$$\Rightarrow \mathcal{A}_f \approx 0, \quad \mathcal{S}_f \approx \sin 2(\phi_1 + \phi) = -\sin 2\phi'$$



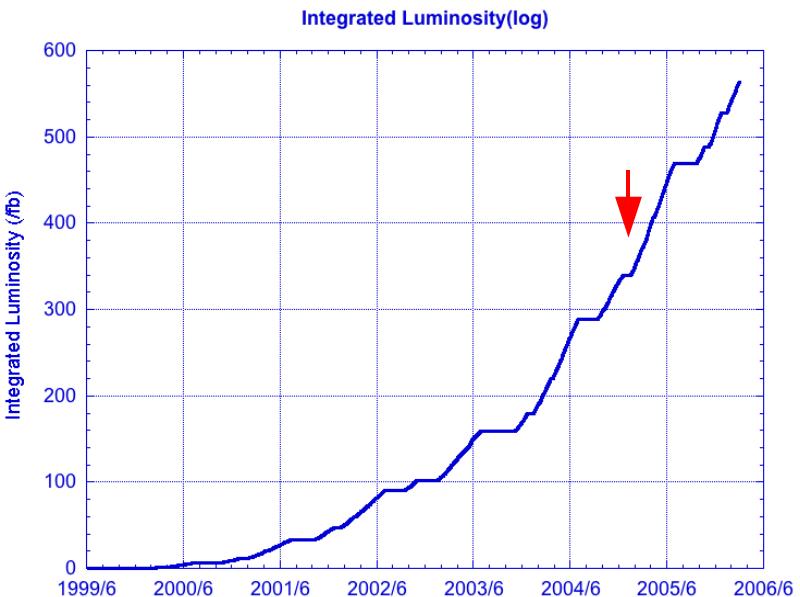


Belle at KEKB



$e^+e^- \rightarrow Y(4S) \rightarrow \bar{B}B$

3.5 GeV on 8.0 GeV



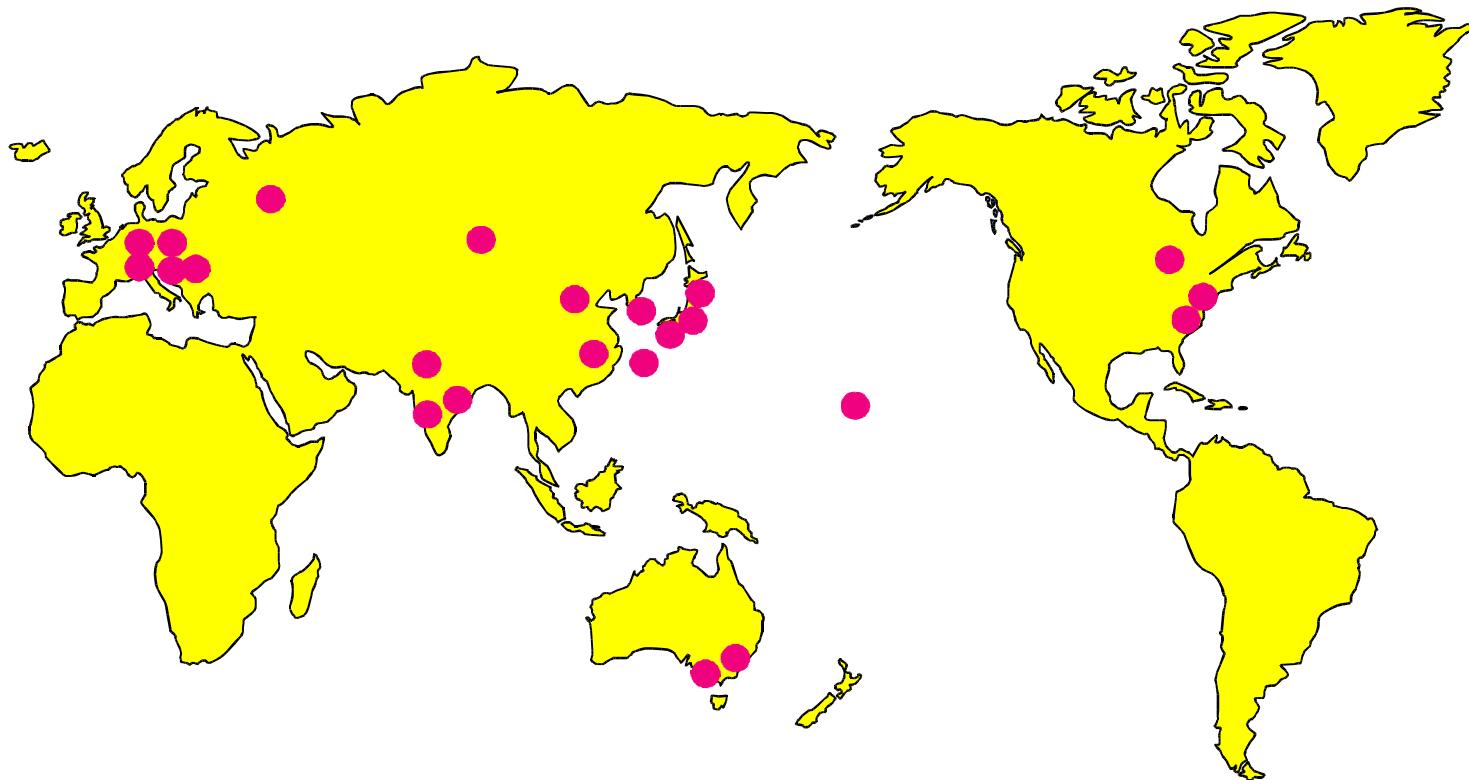
$$\int L dt = 558 \text{ fb}^{-1} \text{ today}$$

$$L_{peak} (\text{max}) = 1.63 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$$

357 fb^{-1} on resonance (386M $\bar{B}B$)
analyzed thus far



The Belle Collaboration

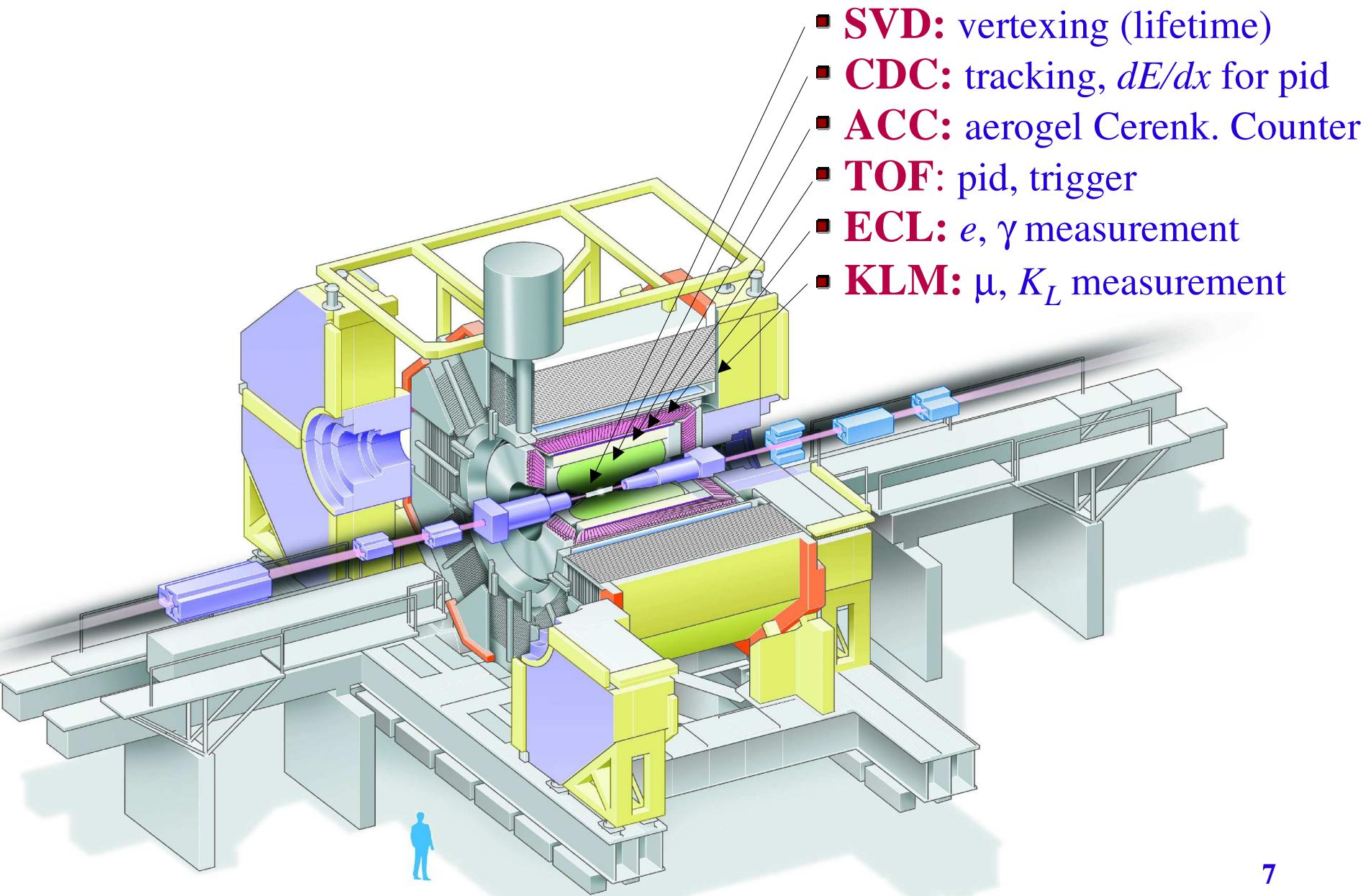


About 400 people from 59 institutions, many countries

USA: Cincinnati
Hawaii
Princeton
Virginia Tech



The Belle detector



1) $B \rightarrow f$ selection:

$$m_{bc} = \sqrt{(E_{beam}^*)^2 - (p_B^*)^2}$$

$$\Delta E = E_B^* - E_{beam}^*$$

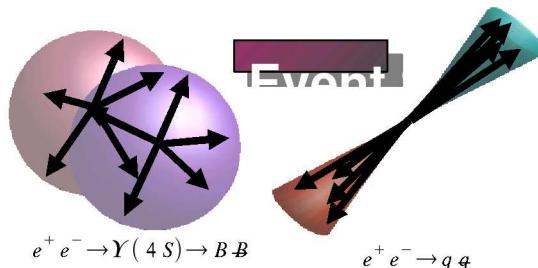
(e.g., for $B \rightarrow \pi^+ \pi^-$:
 $5.271 < m_{bc} < 5.287 \text{ GeV}/c^2$
 $|\Delta E| < 0.064 \text{ GeV})$

2) Flavor tagging:

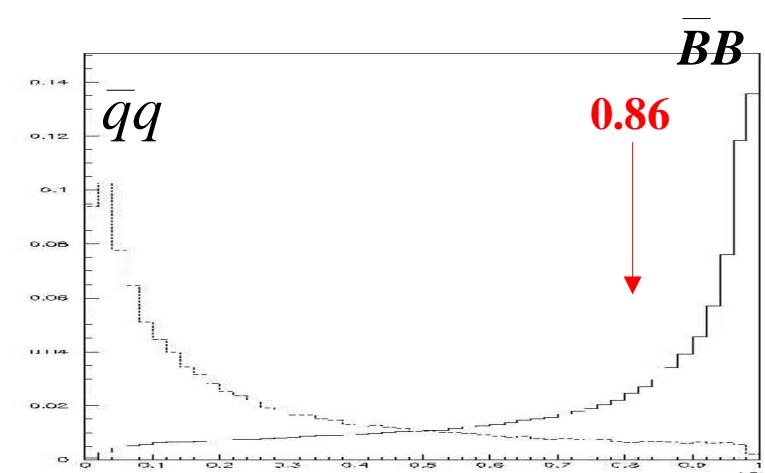
mainly K^\pm, μ^\pm, e^\pm

output: $q = \pm 1, \text{ quality } r = 0-1$

3) Continuum suppression:



$$KLR \equiv \frac{\mathcal{L}_{B\bar{B}}}{(\mathcal{L}_{B\bar{B}} + \mathcal{L}_{q\bar{q}})}$$



4) Vertexing and Δt fit

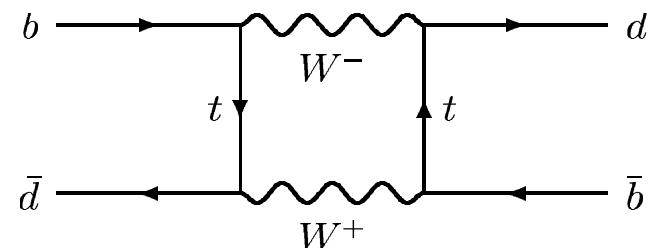
KLR

Measurement of $\sin(2\phi_1)$ with $B^0 \rightarrow J/\psi K^0$

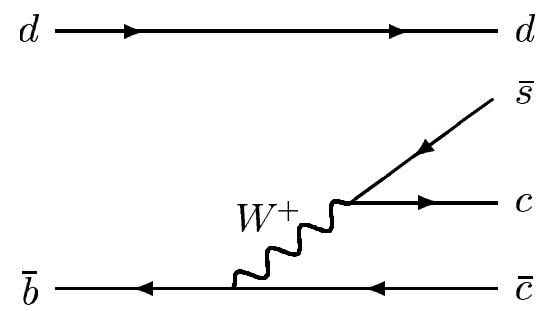
$$\begin{aligned}
 \lambda &= \sqrt{\frac{M_{12}^*}{M_{12}}} \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} = - \left(\frac{V_{td} V_{tb}^*}{V_{td}^* V_{tb}} \right) \left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \right) \left(\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \right) \\
 &= - \frac{V_{td} V_{tb}^* V_{cb} V_{cd}^*}{V_{td}^* V_{tb} V_{cb}^* V_{cd}} \\
 &= - \frac{-V_{cb} V_{cd}^* / (V_{td}^* V_{tb})}{-V_{cb}^* V_{cd} / (V_{td} V_{tb}^*)} \\
 &= - \frac{|\mathcal{M}| e^{-i\phi_1}}{|\mathcal{M}| e^{i\phi_1}} \\
 &= -e^{-2i\phi_1}
 \end{aligned}$$

$$\Rightarrow \mathcal{A}_{(J/\psi K^0)} = 0 \quad \mathcal{S}_{(J/\psi K^0)} = \sin(2\phi_1)$$

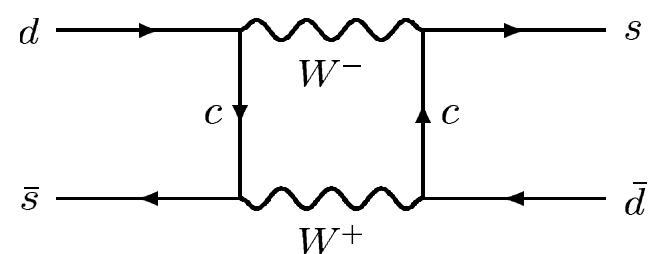
\bar{B}^0 - B^0 oscillation:



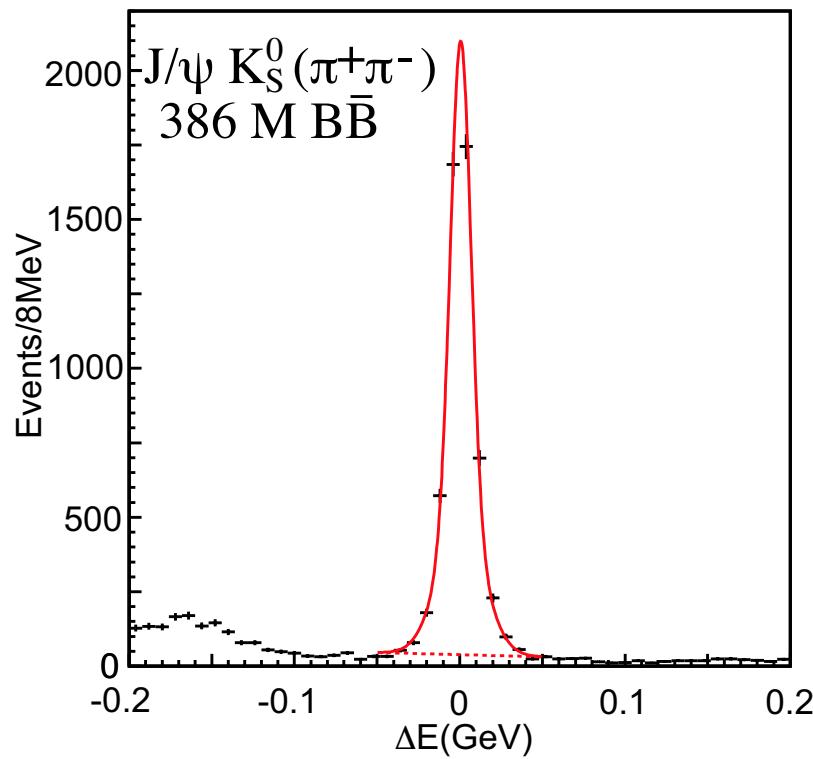
Tree:



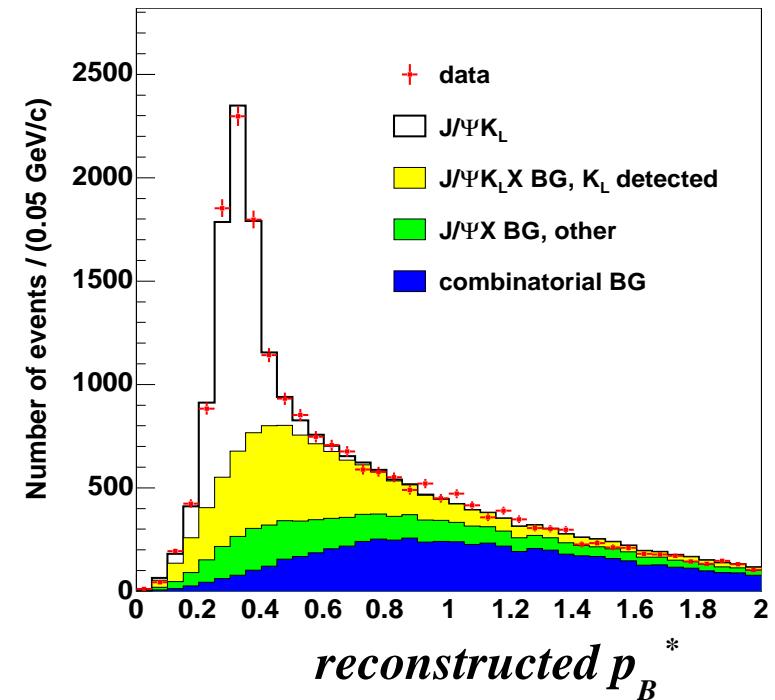
\bar{K}^0 - K^0 oscillation:



357 fb⁻¹:

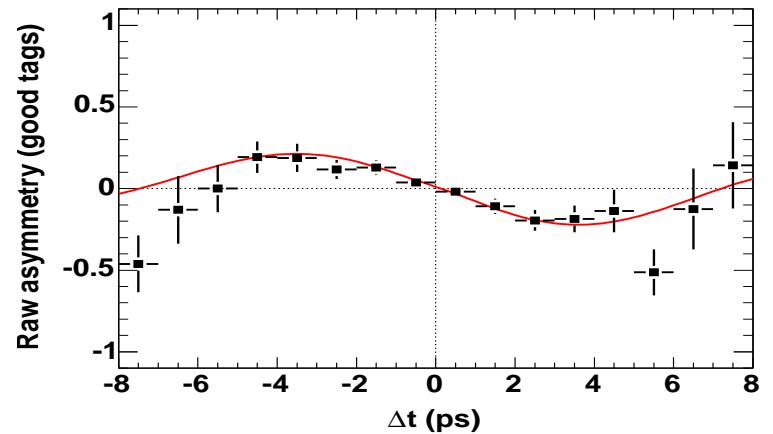
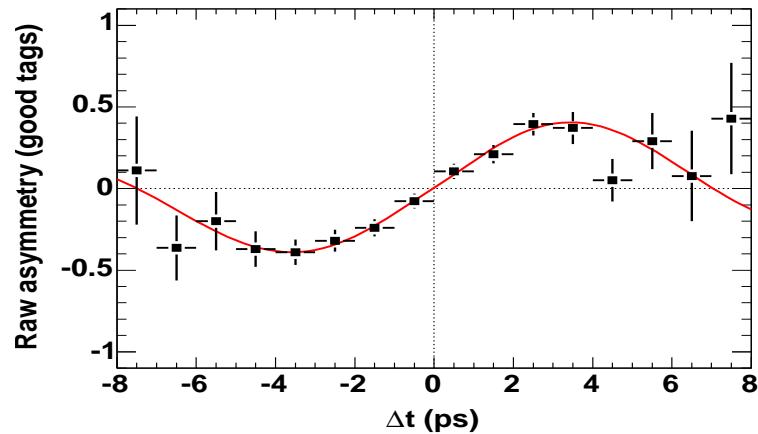
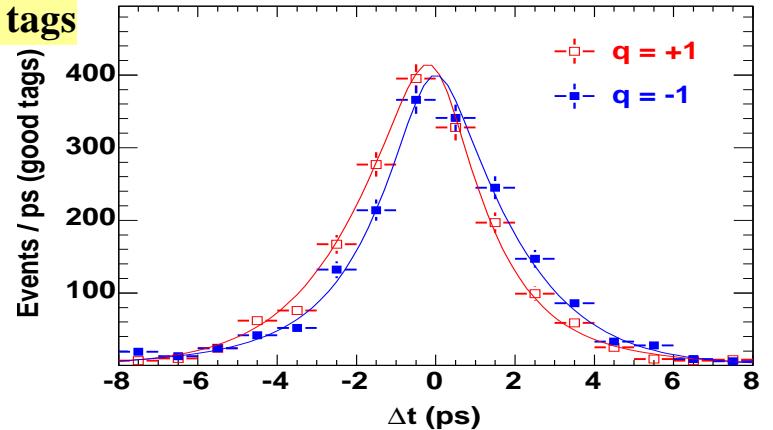
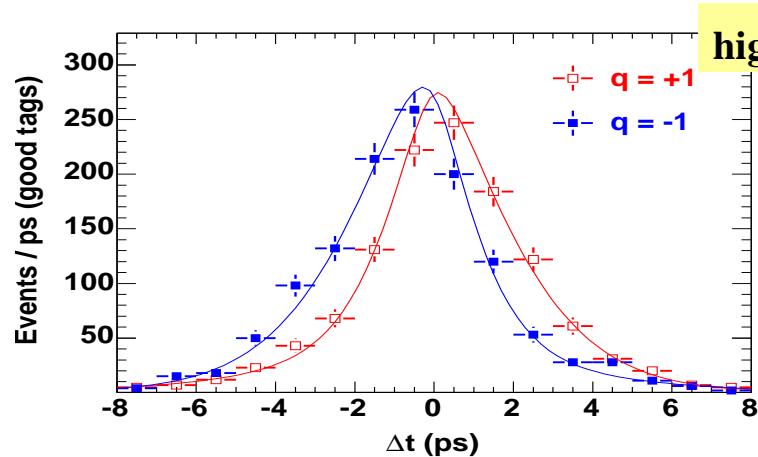


$B^0 \rightarrow J/\psi K_S$ ($CP = -1$)
5264 events
98% pure



$B^0 \rightarrow J/\psi K_L$ ($CP = +1$)
4792 events
60% pure

Note: cannot use ΔE because we don't have E_{KL}



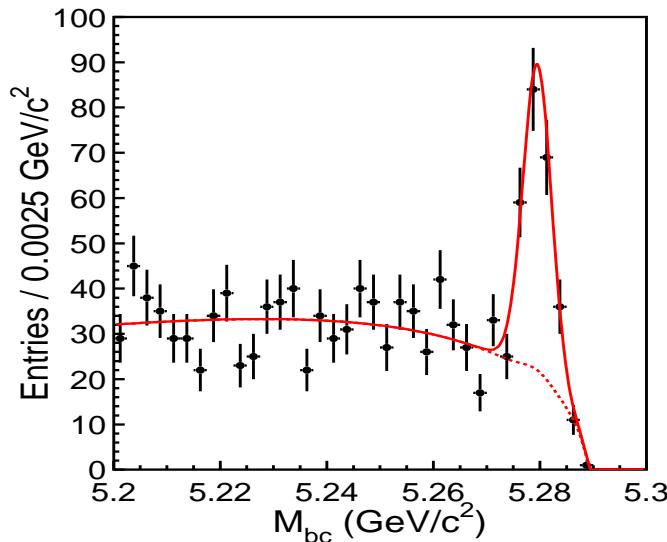
$$\begin{aligned} \sin(2\phi_1) &= 0.652 \pm 0.039 \pm 0.020 \\ \Rightarrow \quad \phi_1 &= (20.3^{+1.7}_{-1.6})^\circ \end{aligned}$$

close to BaBar 210 fb^{-1} :

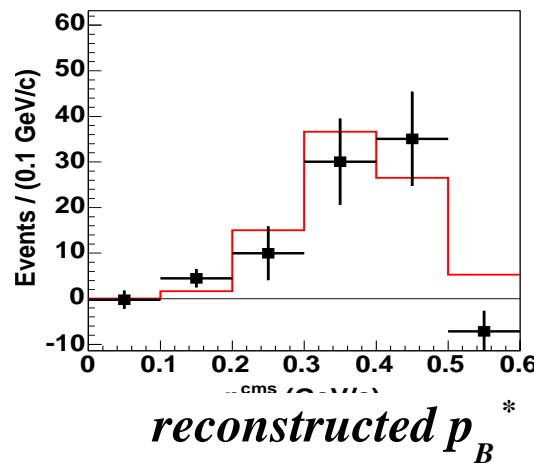
$$\begin{aligned} \sin(2\phi_1) &= 0.722 \pm 0.040 \pm 0.023 \\ |\lambda| &= 0.950 \pm 0.031 \pm 0.013 \end{aligned}$$



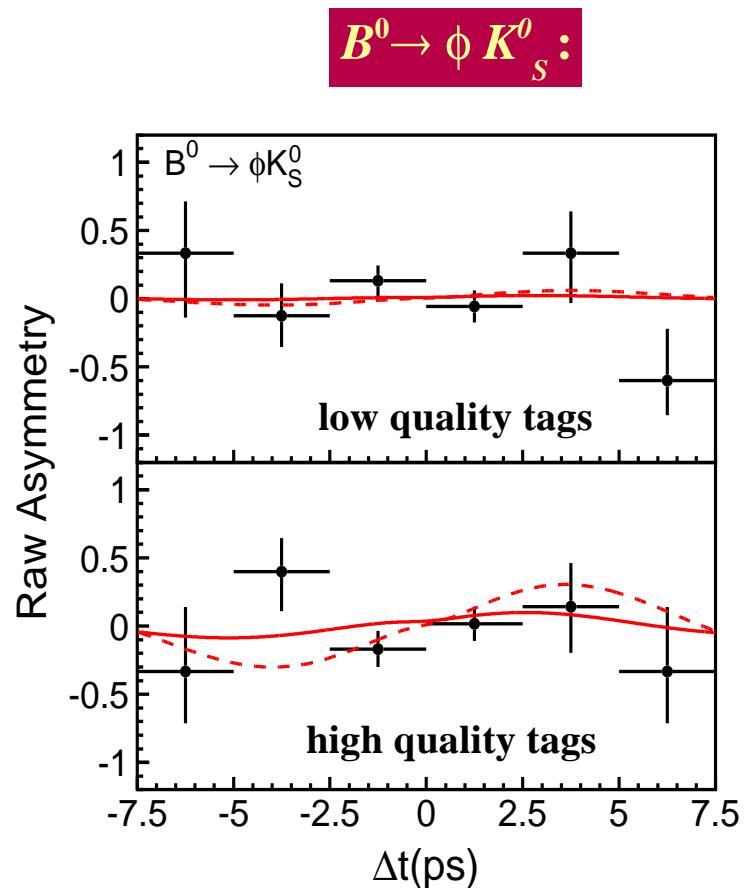
Measurement of $\sin(2\phi_1)$ with $b \rightarrow sss$ (hep-ex/0507037)



$B^0 \rightarrow \phi K_s$ ($CP = -1$)
 $N = 180 \pm 16$
57% pure



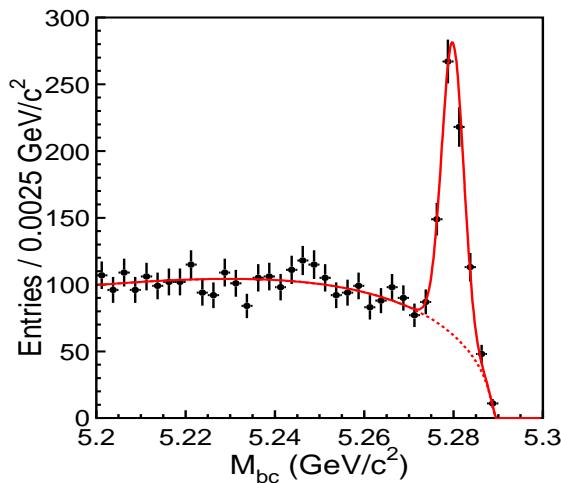
$B^0 \rightarrow \phi K_L$ ($CP = +1$)
 $N = 78 \pm 13$
12% pure



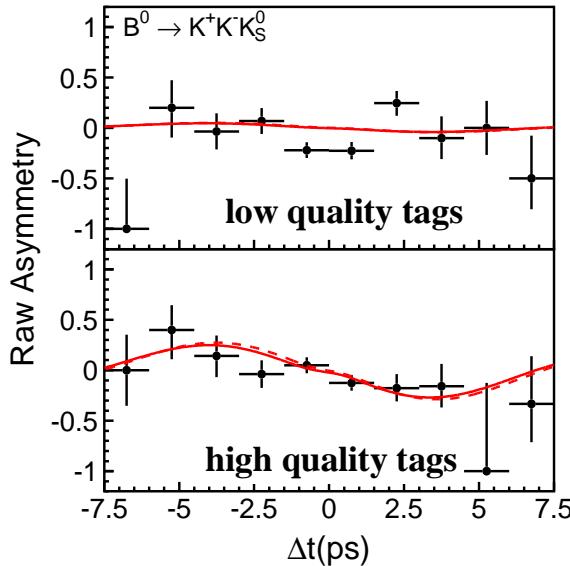
$\sin(2\phi_1) = + 0.44 \pm 0.27 \pm 0.05$
 $A = + 0.14 \pm 0.17 \pm 0.07$



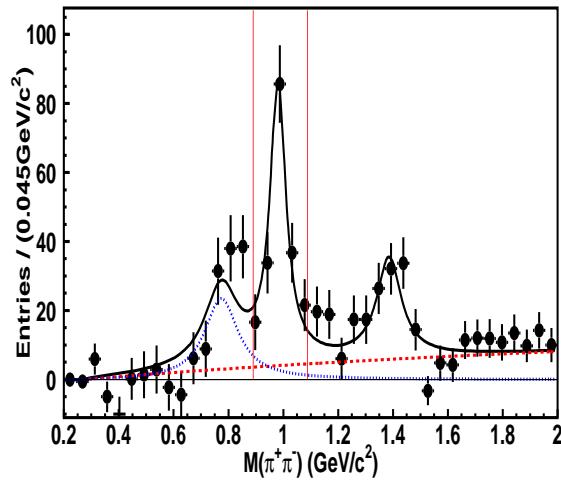
Measurement of $\sin(2\phi_1)$ with $b \rightarrow qqs$ (hep-ex/0507037)



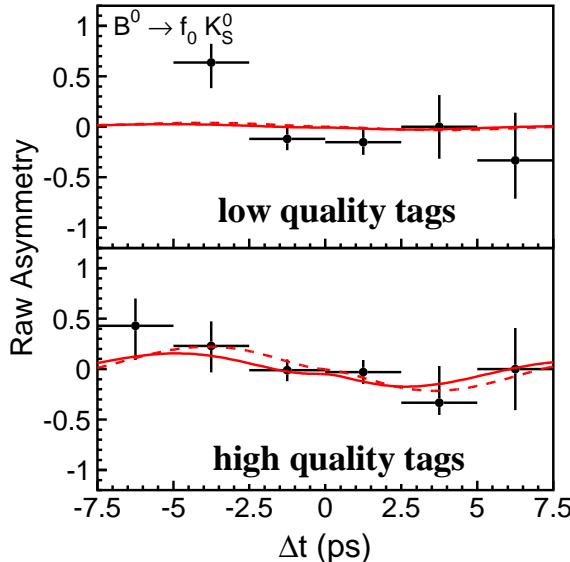
$B^0 \rightarrow K^+ K^- K_S$
 $(CP = +1 \text{ mostly})$
 $N = 536 \pm 29$
 55% pure



$\sin(2\phi_1) =$
 $-0.52 \pm 0.16 \pm 0.03$
 $A =$
 $-0.06 \pm 0.11 \pm 0.07$



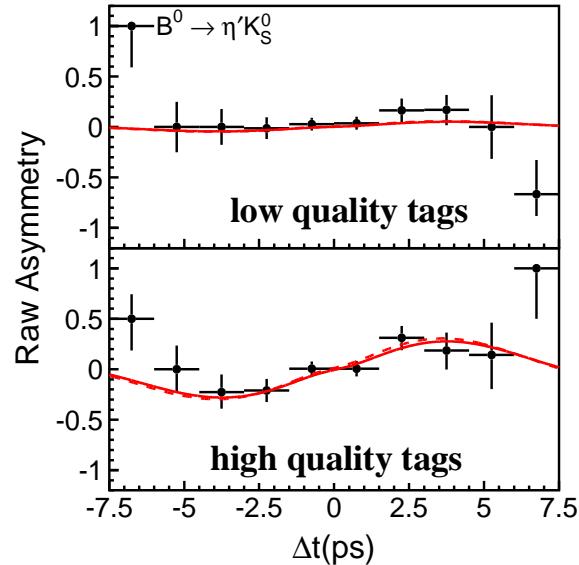
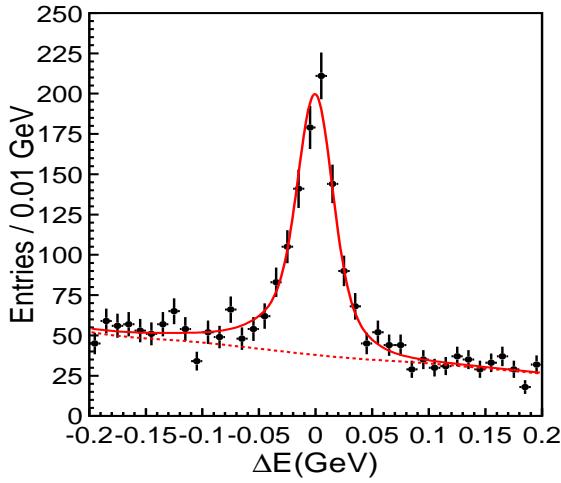
$B^0 \rightarrow f_0(980) K_S$
 $(CP = +1)$
 $N = 145 \pm 16$
 47% pure



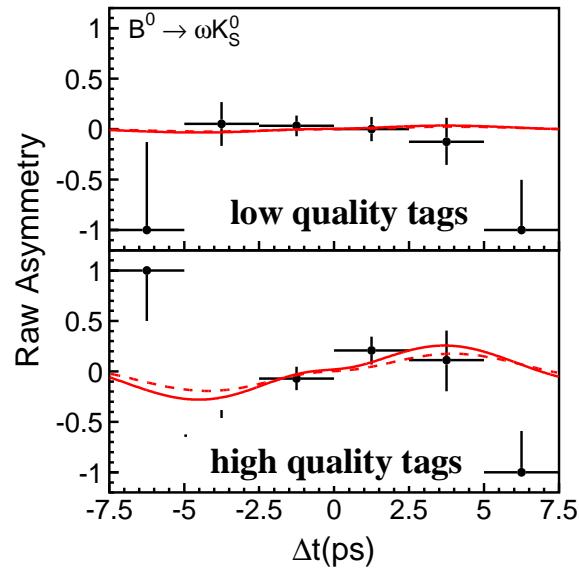
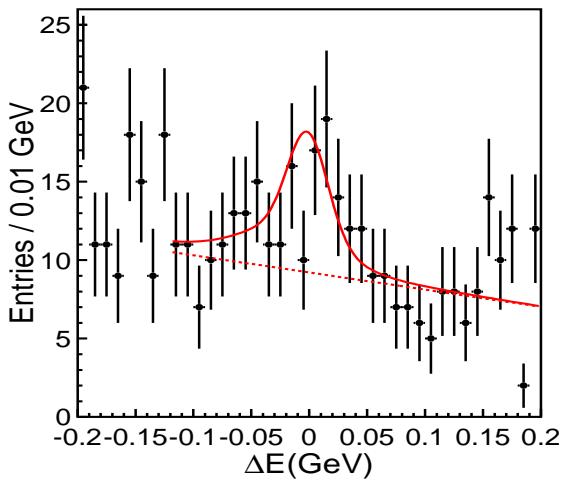
$\sin(2\phi_1) =$
 $-0.47 \pm 0.36 \pm 0.08$
 $A =$
 $-0.23 \pm 0.23 \pm 0.13$



Measurement of $\sin(2\phi_1)$ with $b \rightarrow qqs$ (hep-ex/0507037)



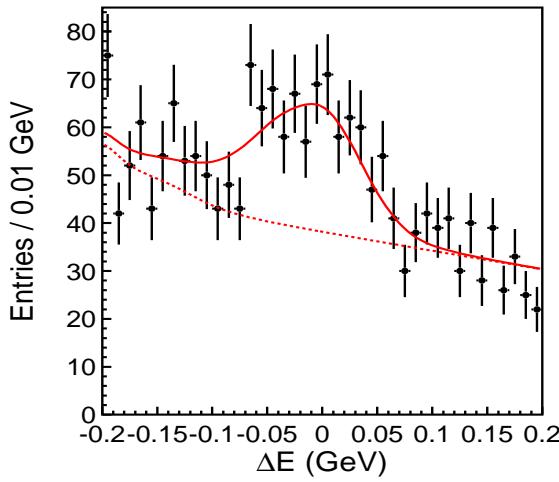
$$\begin{aligned} \sin(2\phi_1) &= \\ &+ 0.62 \pm 0.12 \pm 0.04 \\ A &= \\ &- 0.04 \pm 0.08 \pm 0.06 \end{aligned}$$



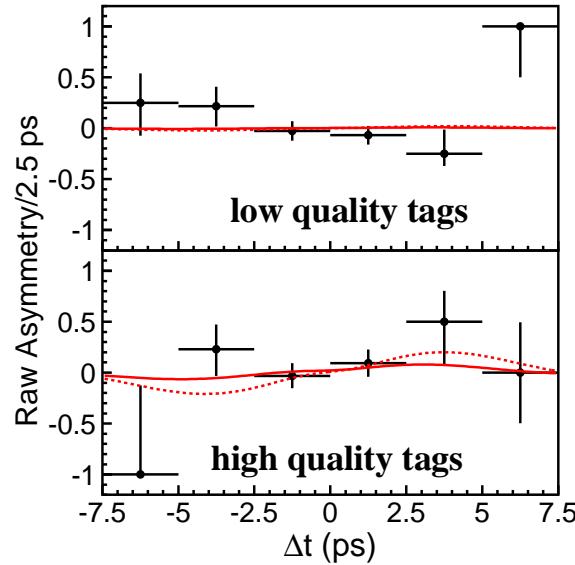
$$\begin{aligned} \sin(2\phi_1) &= \\ &+ 0.95 \pm 0.53 {}^{+0.12}_{-0.15} \\ A &= \\ &+ 0.19 \pm 0.39 \pm 0.13 \end{aligned}$$



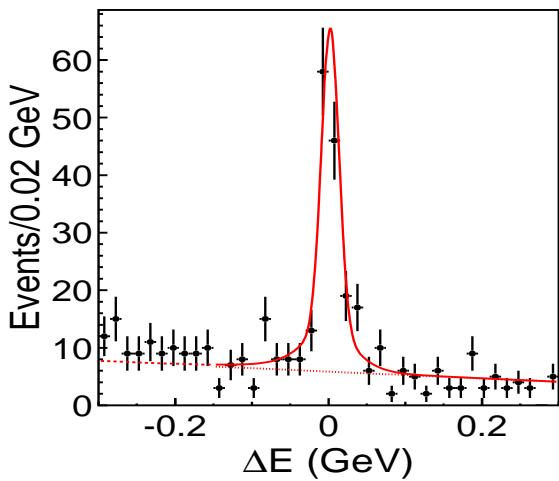
Measurement of $\sin(2\phi_1)$ with $b \rightarrow qqs$ (hep-ex/0507037)



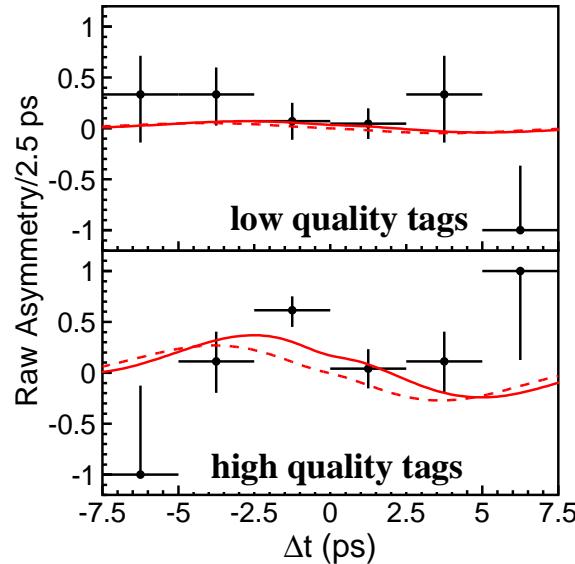
$B^0 \rightarrow \pi^0 K_s$
 $(CP = -1)$
 $N = 344 \pm 30$
25% pure



$\sin(2\phi_1) =$
 $+ 0.22 \pm 0.47 \pm 0.08$
 $A =$
 $+ 0.11 \pm 0.18 \pm 0.08$

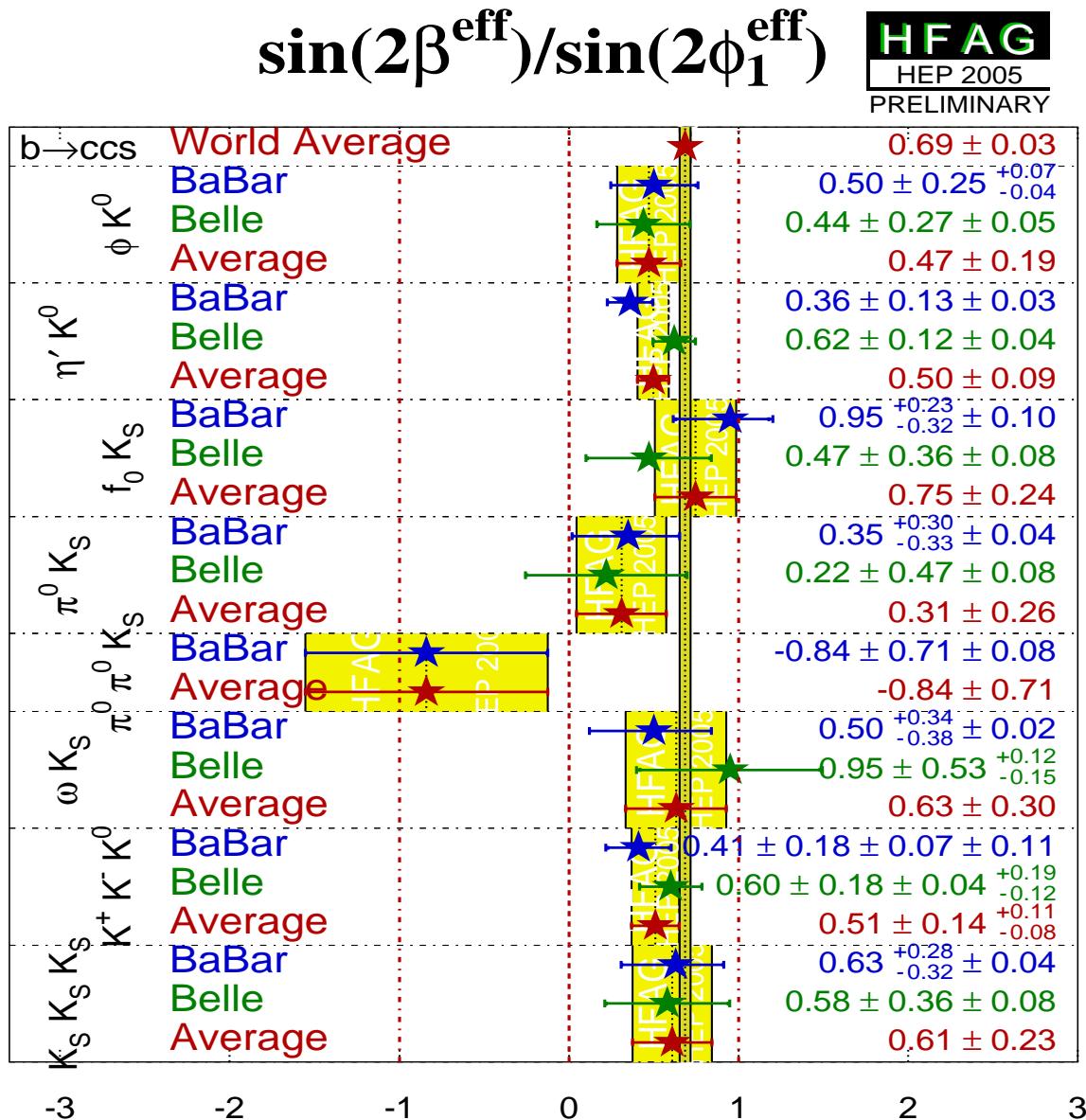


$B^0 \rightarrow K_s K_s K_s$
 $(CP = +1)$
 $N = 105 \pm 12$
64% pure



$\sin(2\phi_1) =$
 $- 0.58 \pm 0.36 \pm 0.08$
 $A =$
 $+ 0.50 \pm 0.23 \pm 0.06$

Measurement of $\sin(2\phi_1)$ summary



⇒ modulu final-state interactions
 [see Cheng, Chua, Soni,
 PRD 72, 014006 (2005)],
 results appear consistent
 with SM

HFAG Belle +BaBar:

$$\phi_1 = (21.7^{+1.3}_{-1.2})^\circ$$

$$\text{or } (68.3^{+1.2}_{-1.3})^\circ$$

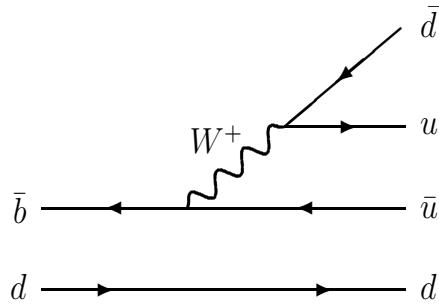
(excluded by $B^0 \rightarrow J/\psi K^*$)

Measurement of $\sin(2\phi_2)$ with $B^0 \rightarrow \pi^+ \pi^-$

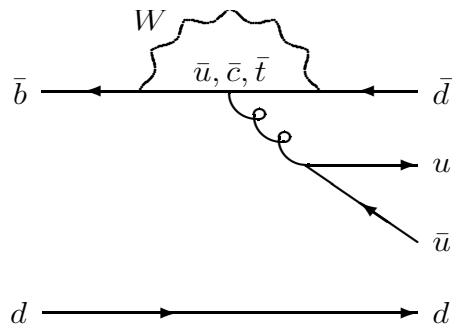
$$\begin{aligned}
 \lambda &= \sqrt{\frac{M_{12}^*}{M_{12}}} \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} = + \left(\frac{V_{td} V_{tb}^*}{V_{td}^* V_{tb}} \right) \left(\frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}} \right) \\
 &= \frac{-V_{tb}^* V_{td} / (V_{ub}^* V_{ud})}{-V_{tb} V_{td}^* / (V_{ub} V_{ud}^*)} \\
 &= \frac{|\mathcal{M}'| e^{i\phi_2}}{|\mathcal{M}'| e^{-i\phi_2}} \\
 &= e^{2i\phi_2}
 \end{aligned}$$

$$\Rightarrow \mathcal{A}_{\pi\pi} = 0 \quad \mathcal{S}_{\pi\pi} = \sin(2\phi_2)$$

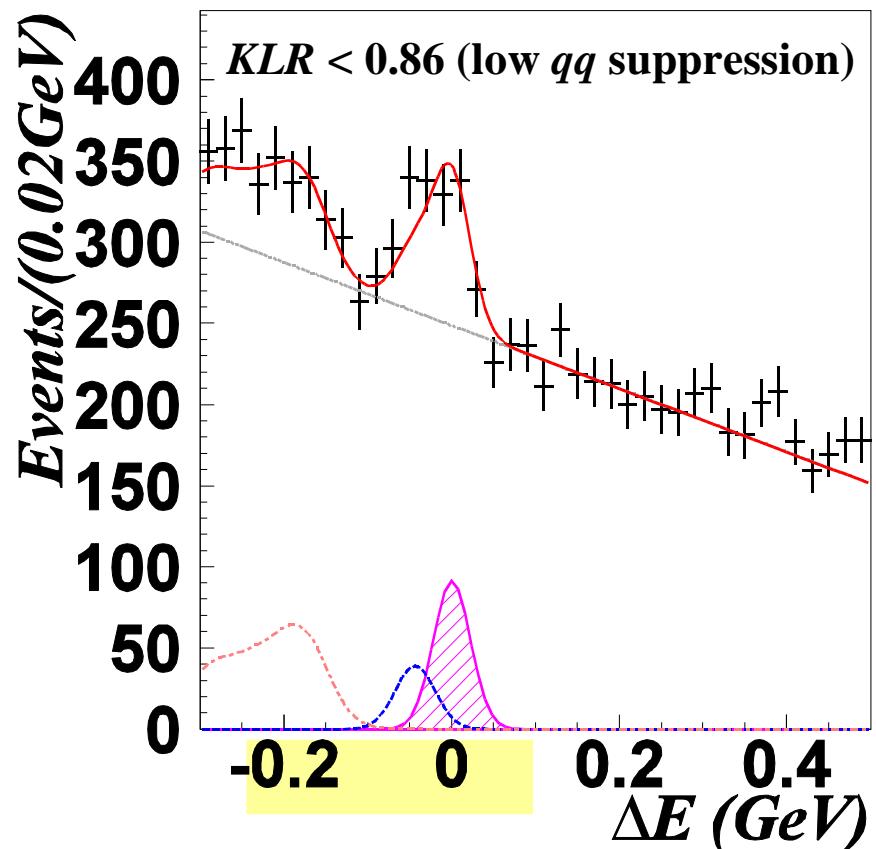
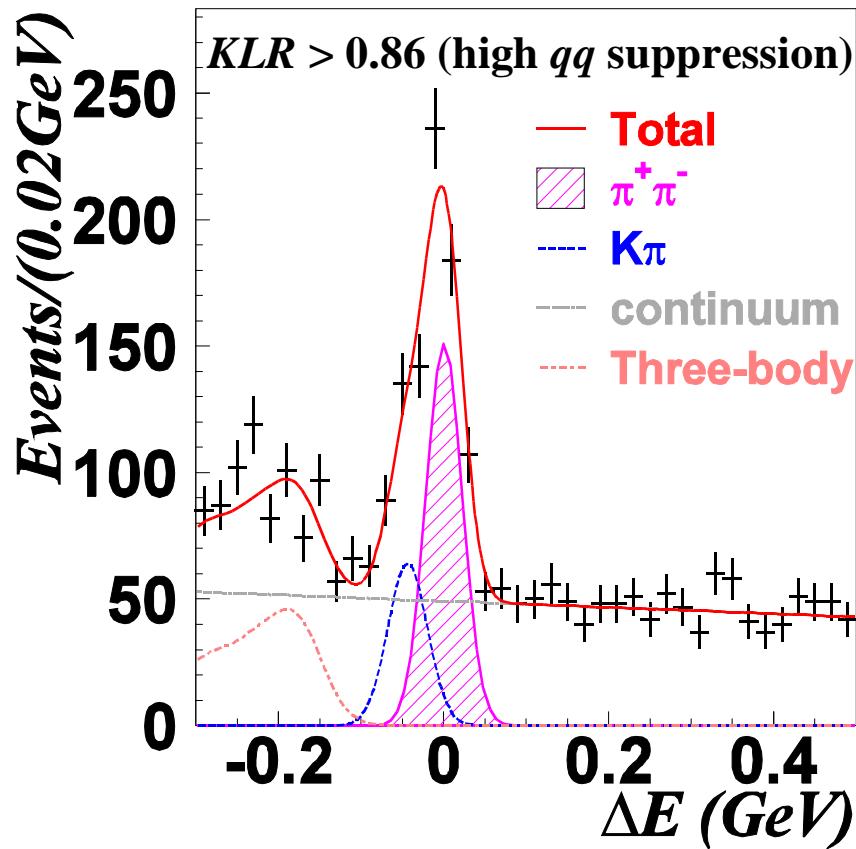
Tree:



Penguin:



...if no penguin. But there is a penguin contribution, which “breaks” these equalities



$$N_{\pi\pi} = 415 \pm 13$$

$$N_{\pi\pi} = 251 \pm 8$$

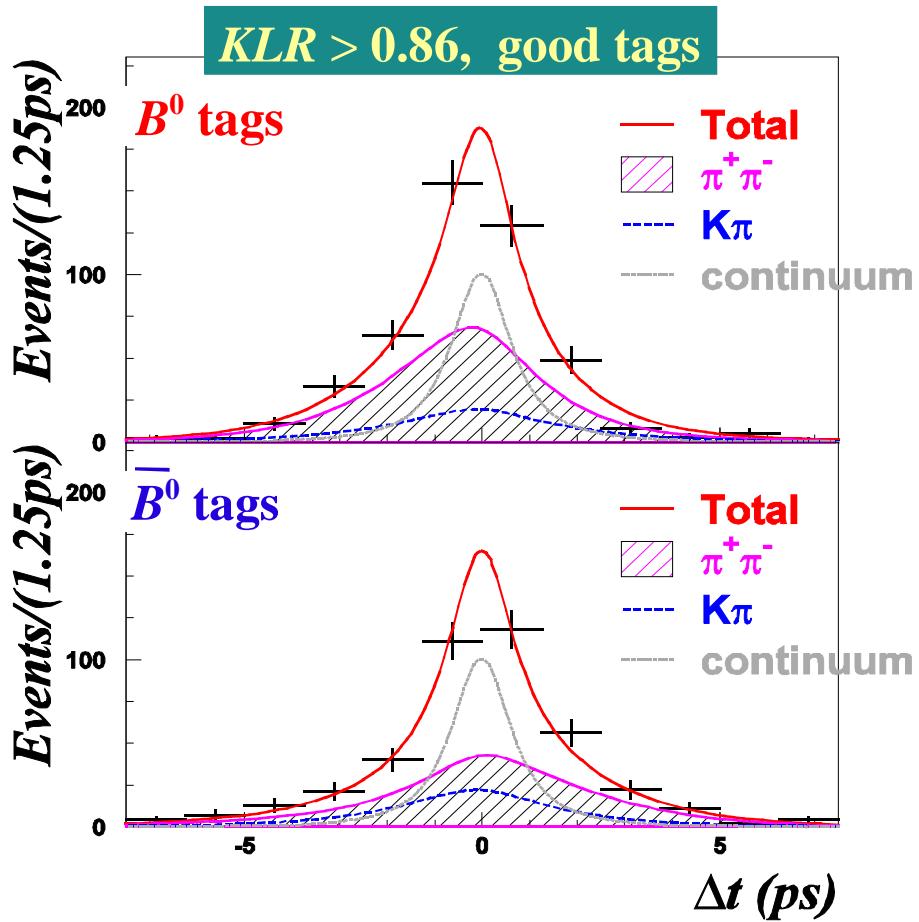


Maximum likelihood fit to Δt ($B^0 \rightarrow \pi^+ \pi^-$)

$$\begin{aligned} \mathcal{L}_i = & \int [f_{\pi\pi} P_{\pi\pi}(\Delta t') + f_{K\pi} P_{K\pi}(\Delta t')] \cdot R_{hh}(\Delta t_i - \Delta t') \\ & + f_{q\bar{q}} P_{q\bar{q}}(\Delta t') \cdot R_{q\bar{q}}(\Delta t_i - \Delta t') dt' \end{aligned}$$

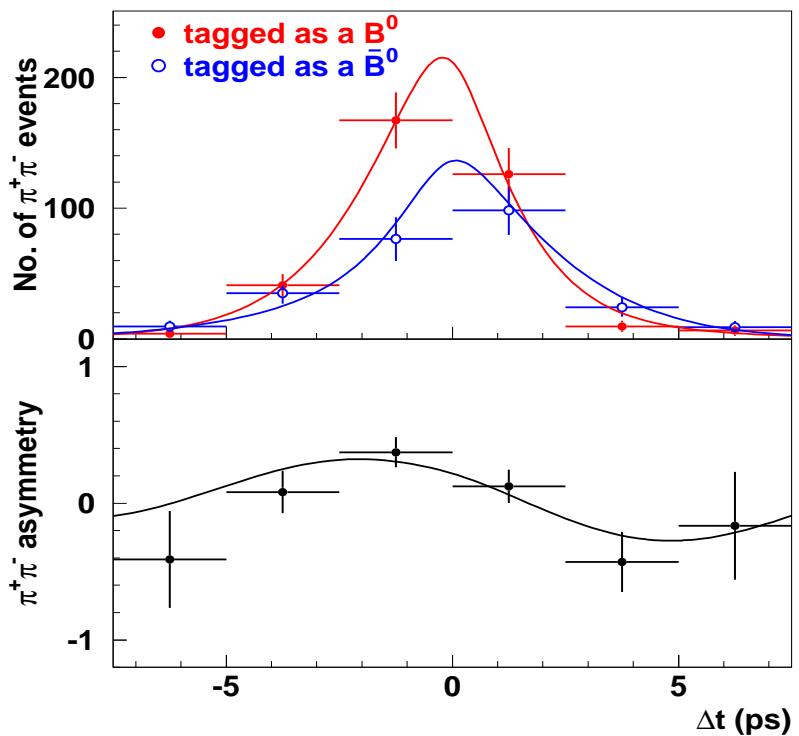
$$\begin{aligned} P_{B^0 \rightarrow \pi\pi}^{(\ell)} &= \frac{e^{-|\Delta t|/\tau_B}}{\mathcal{N}} \left\{ 1 + q(1 - 2\omega_\ell) [\mathcal{A}_{\pi\pi} \cos(\Delta m \Delta t) + \mathcal{S}_{\pi\pi} \sin(\Delta m \Delta t)] \right\} \\ P_{K\pi} &= \frac{e^{-|\Delta t|/\tau_B}}{4\tau_B} \left\{ 1 + q(1 - 2\omega_\ell) \mathcal{A}_{K\pi}^{\text{eff}} \cos(\Delta m \Delta t) \right\} \quad (\mathcal{A}_{K\pi} = -0.109 \pm 0.019) \\ P_{q\bar{q}} &= f \frac{e^{-|\Delta t|/\tau_{q\bar{q}}}}{2\tau_{q\bar{q}}} + (1 - f) \delta(\Delta t), \end{aligned}$$

$$f_{\pi\pi} = \frac{F_{\pi\pi}(\Delta E, M_{bc}) \cdot f_\ell(\pi\pi)}{[F_{\pi\pi}(\Delta E, M_{bc}) + F_{K\pi}(\Delta E, M_{bc})] \cdot f_\ell(\pi\pi) + F_{q\bar{q}}(\Delta E, M_{bc}) \cdot f_\ell(q\bar{q})}$$



$$A_{\pi\pi} = 0.56^{+0.11}_{-0.12} \quad S_{\pi\pi} = -0.67 \pm 0.16$$

$m_{bc} - \Delta E$ 2D fit for
event yields in bins
of Δt :





Systematic Uncertainties

Uncertainty	$A_{\pi\pi}$	$S_{\pi\pi}$
Wrong tag fraction	± 0.01	± 0.01
τ_B , Δm , $A_{K\pi}$	± 0.01	< 0.01
Resolution function	± 0.01	± 0.04
Background Δt shape	< 0.01	< 0.01
Background fractions	± 0.04	± 0.02
Fit bias	± 0.01	± 0.01
Vertexing	$+0.03$ -0.01	± 0.04
Tag side interference	$+0.02$ -0.04	± 0.01
Total	± 0.06	± 0.06

← includes uncertainty
in final state radiation

← O. Long *et al.*,
PRD 68, 034010 (2003)



Constraints upon ϕ_2 (α) and $|P/T|$

Gronau and Rosner,
PRD 65, 093012, 2002:

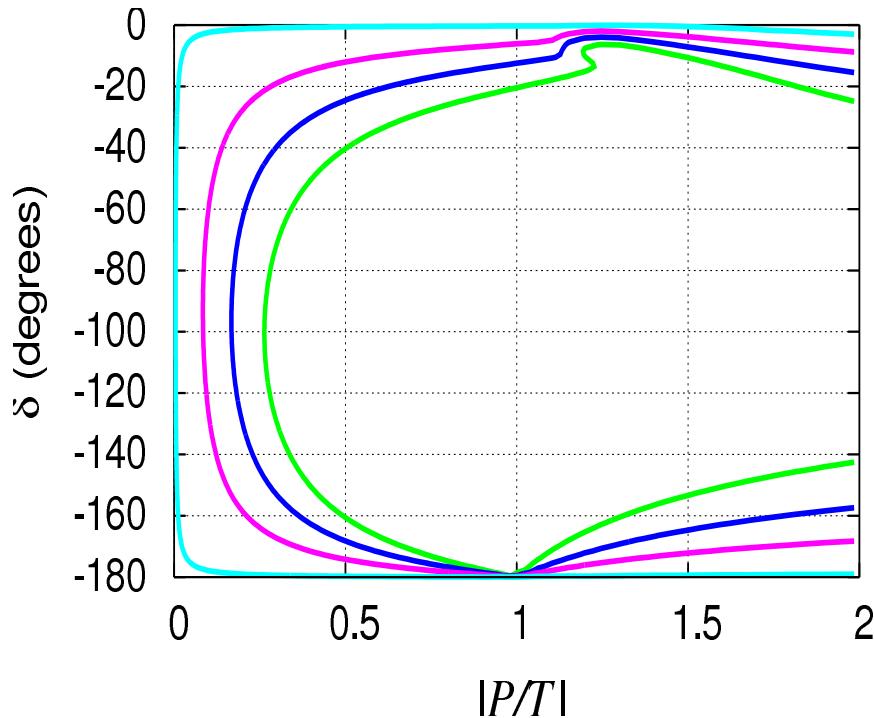
$$\begin{aligned}
 A(B^0 \rightarrow \pi^+ \pi^-) &= -(|T| e^{i\delta_T} e^{i\phi_3} + |P| e^{i\delta_P}) \\
 A(\bar{B}^0 \rightarrow \pi^+ \pi^-) &= -(|T| e^{i\delta_T} e^{-i\phi_3} + |P| e^{i\delta_P}) \\
 \Rightarrow \lambda_{\pi\pi} \equiv \frac{q}{p} \frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}} &= e^{i\phi_2} \frac{1 + |P/T| e^{i(\delta + \phi_3)}}{1 + |P/T| e^{i(\delta - \phi_3)}} \\
 &\quad (\delta \equiv \delta_P - \delta_T)
 \end{aligned}$$

Take $\phi_1 = 0.725 \pm 0.037$
 $\Rightarrow 2$ constraints &
 3 unknowns
 $(\phi_2, \delta, |P/T|)$

$$\begin{aligned}
 A_{\pi\pi} &\equiv \frac{|\lambda|^2 - 1}{|\lambda|^2 + 1} = \frac{-2|P/T| \sin(\phi_1 + \phi_2) \sin \delta}{1 - 2|P/T| \cos(\phi_1 + \phi_2) \cos \delta + |P/T|^2} \\
 S_{\pi\pi} &\equiv \frac{2Im\lambda}{|\lambda|^2 + 1} \\
 &= \frac{2|P/T| \sin(\phi_1 - \phi_2) \cos \delta + \sin 2\phi_2 - |P/T|^2 \sin 2\phi_1}{1 - 2|P/T| \cos(\phi_1 + \phi_2) \cos \delta + |P/T|^2}
 \end{aligned}$$



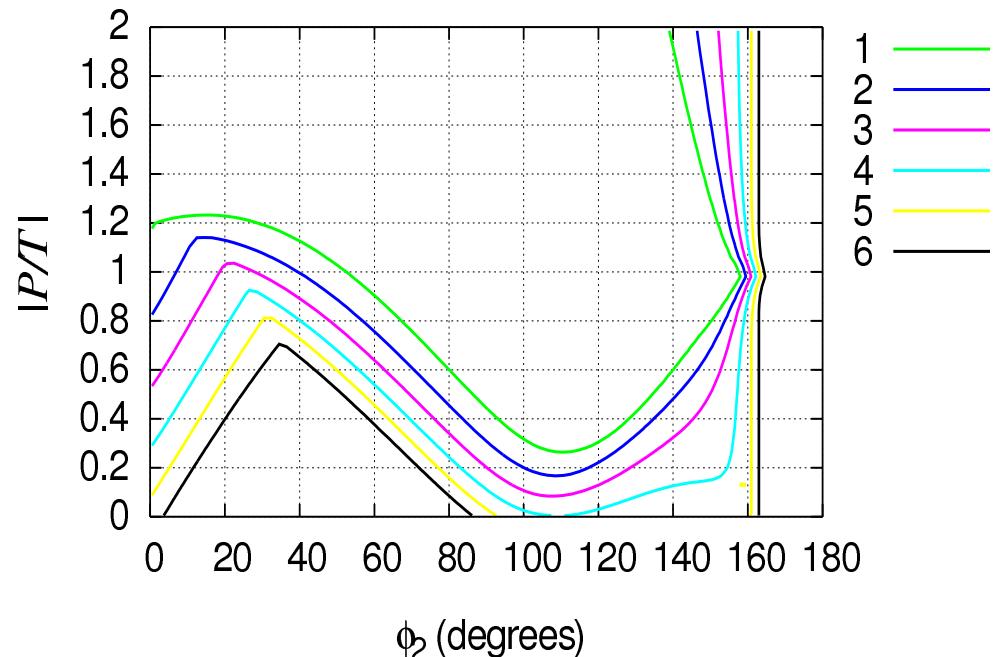
Constraints upon ϕ_2 (α) and $|P/T|$ cont'd



For $|P/T| = 0.6$ (for example)
 $72^\circ < \phi_2 < 146^\circ$ (95% CL)

1
2
3
4

For any $|P/T|$
 $\delta < -4^\circ$ (95% CL)
For any δ
 $|P/T| > 0.17$ (95% CL)



$SU(2)$ isospin analysis:

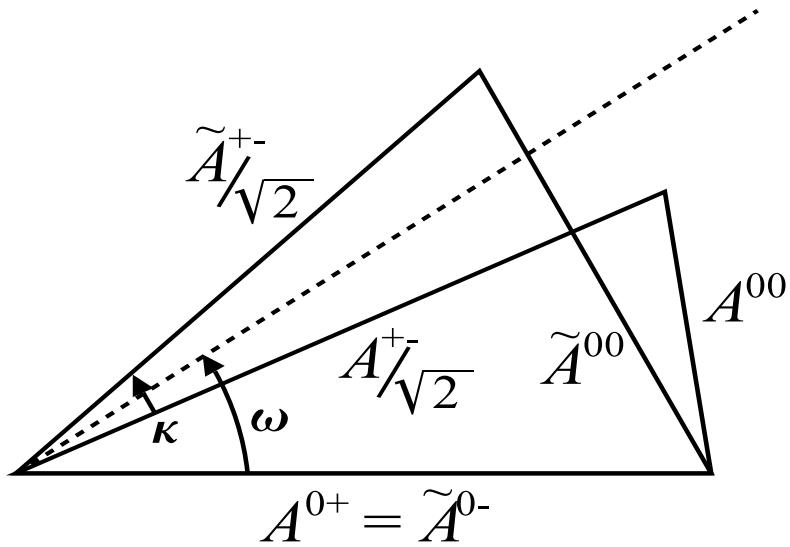
Gronau and London,
PRL 65, 3381 (1990)

$$\frac{A(B^0 \rightarrow \pi^+ \pi^-)}{\sqrt{2}} + A(B^0 \rightarrow \pi^0 \pi^0) = A(B^+ \rightarrow \pi^+ \pi^0)$$

$$\frac{A(\bar{B}^0 \rightarrow \pi^+ \pi^-)}{\sqrt{2}} + A(\bar{B}^0 \rightarrow \pi^0 \pi^0) = A(B^- \rightarrow \pi^- \pi^0)$$

6 param. + 6 observables \Rightarrow all determined

Recent measurements (253 fb^{-1}) of
 $\bar{B}^0(B^0) \rightarrow \pi^0 \pi^0$ now make this possible



$$|A_{\text{th}}^{+-}| = \sqrt{a^{+-}(1 - \mathcal{A}_{\pi\pi})}$$

$$|\bar{A}_{\text{th}}^{+-}| = \sqrt{a^{+-}(1 + \mathcal{A}_{\pi\pi})}$$

$$|A_{\text{th}}^{0+}| = |A_{\text{th}}^{0+}| = \sqrt{a^{0+}}$$

$$|A_{\text{th}}^{00}|^2 = \frac{|A_{\text{th}}^{+-}|^2}{2} + |A_{\text{th}}^{0+}|^2 - \sqrt{2} |A_{\text{th}}^{+-}| |A_{\text{th}}^{0+}| \cos(\omega - \kappa/2)$$

$$|\bar{A}_{\text{th}}^{00}|^2 = \frac{|\bar{A}_{\text{th}}^{+-}|^2}{2} + |A_{\text{th}}^{0+}|^2 - \sqrt{2} |\bar{A}_{\text{th}}^{+-}| |A_{\text{th}}^{0+}| \cos(\omega + \kappa/2)$$

$$B_{\text{th}}^{\pi^+ \pi^-} = \left(|A_{\text{th}}^{+-}|^2 + |\bar{A}_{\text{th}}^{+-}|^2 \right) / 2 = a^{+-}$$

$$B_{\text{th}}^{\pi^0 \pi^0} = \left(|A_{\text{th}}^{00}|^2 + |\bar{A}_{\text{th}}^{00}|^2 \right) / 2$$

$$B_{\text{th}}^{\pi^0 \pi^+} = |A_{\text{th}}^{0+}|^2 (\tau_{B^\pm} / \tau_{B^0}) = a^{+0} \cdot (\tau_{B^\pm} / \tau_{B^0})$$

$$\mathcal{A}_{\text{th}}^{\pi^0 \pi^0} = \frac{|\bar{A}_{\text{th}}^{00}|^2 - |A_{\text{th}}^{00}|^2}{|\bar{A}_{\text{th}}^{00}|^2 + |A_{\text{th}}^{00}|^2}$$

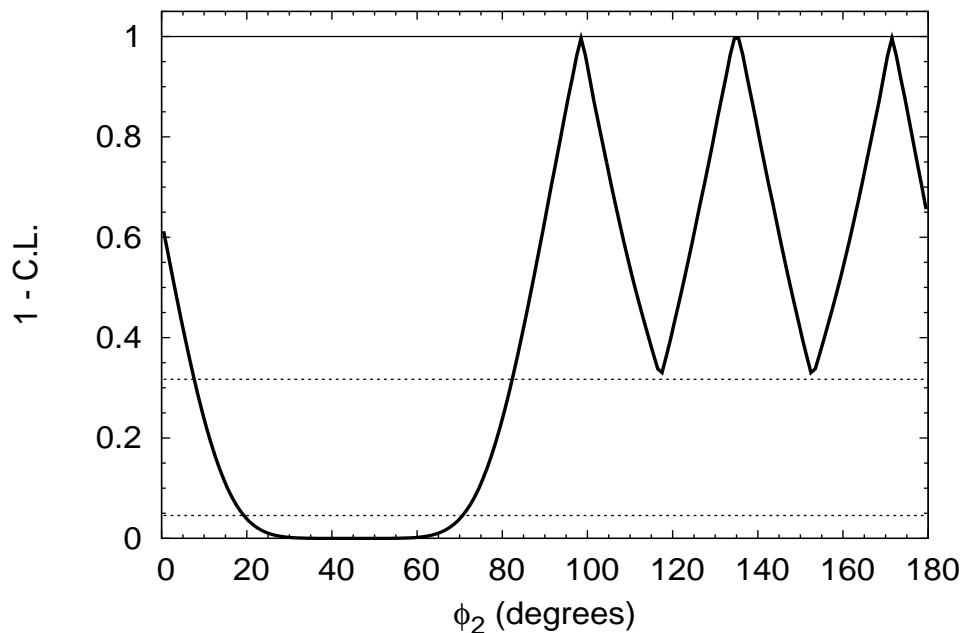
$$\mathcal{A}_{\text{th}}^{\pi^+ \pi^-} = \mathcal{A}_{\pi\pi}$$

$$\mathcal{S}_{\text{th}}^{\pi^+ \pi^-} = \sqrt{1 - \mathcal{A}_{\pi\pi}^2} \sin(2\phi_2 + \kappa)$$

Use HFAG values for $B(\pi^+\pi^-)$, $B(\pi^+\pi^0)$, $B(\pi^0\pi^0)$, $\mathcal{A}(\pi^0\pi^0)$

Calculate χ^2 :

$$\chi^2(\vec{y}) = \sum \frac{(x_{\text{exp}} - x_{\text{th}})^2}{\sigma_{\text{exp}}^2} + \chi^2_{FC}(\mathcal{A}_{\text{th}}^{\pi^+\pi^-}, \mathcal{S}_{\text{th}}^{\pi^+\pi^-})$$



$0^\circ < \phi_2 < 19^\circ$ and $71^\circ < \phi_2 < 180^\circ$
(95% CL)



$\sin(2\phi_2)$ with $B^0 \rightarrow \rho^+ \rho^-$

Advantages:

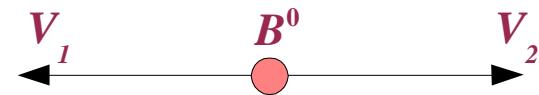
- $CP = +1$ like $\pi^+ \pi^-$
- “large” branching fraction ($> 20 \times 10^{-6}$)
- branching fraction for $B^0 \rightarrow \rho^0 \rho^0 < 1.1 \times 10^{-6}$
 - ⇒ small penguin amplitude
 - ⇒ $A_{\rho\rho} = 0$ and $S_{\rho\rho} = \sin(2\phi_2)$

But complications:

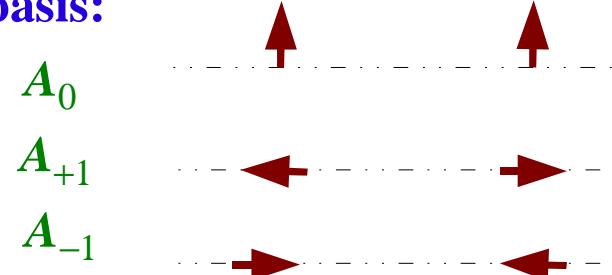
- large $q\bar{q}$ background
- different polarization ⇒ different CP states
- nonresonant contribution
- possible isospin $I=1$ contribution



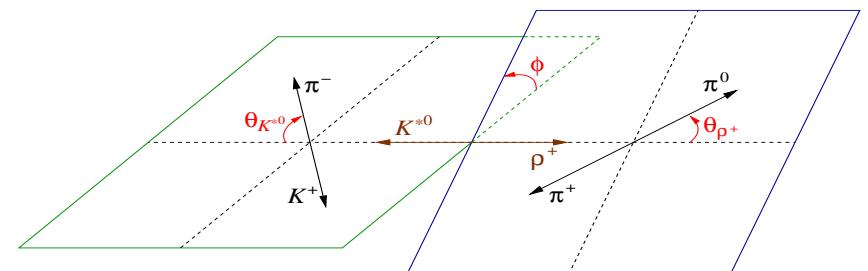
$B^0 \rightarrow VV$ polarization



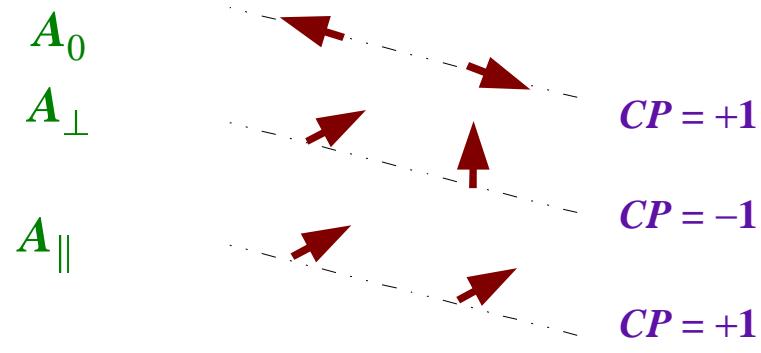
Helicity basis:



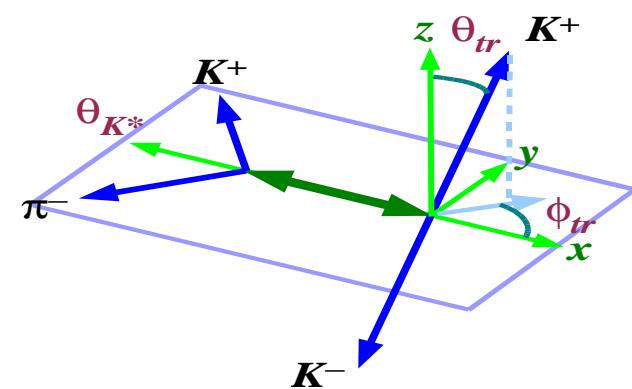
Variables θ_1, θ_2, ϕ :



Transversity basis:



Variables $\theta_K, \theta_{tr}, \phi_{tr}$:

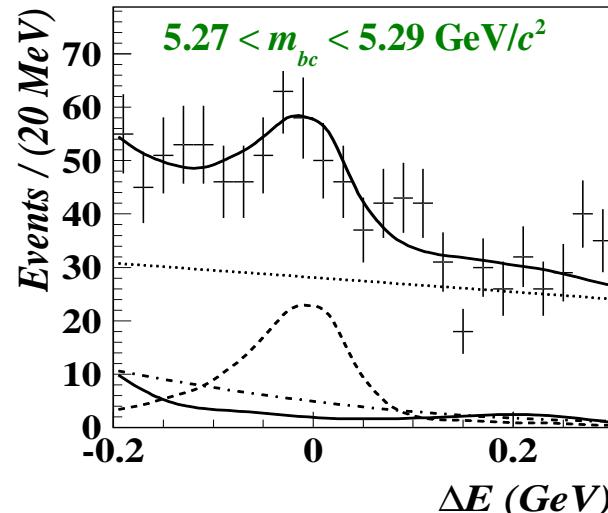
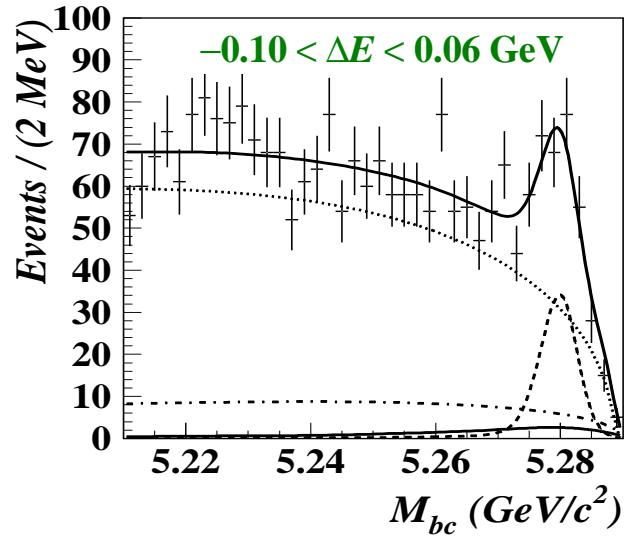


Factorization: $f_L \equiv \frac{|A_0|^2}{|A_0|^2 + |A_{+1}|^2 + |A_{-1}|^2} = \frac{|A_0|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2} \approx 1 - O\left(\frac{m_V^2}{m_B^2}\right) \approx 1$

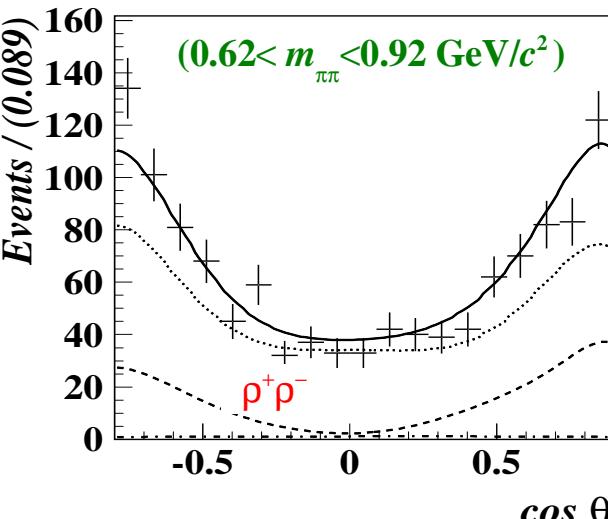
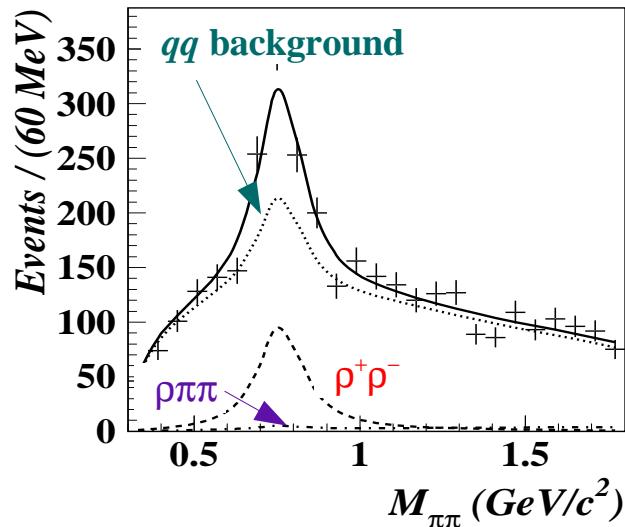
Kagan, PLB 601, 151 (2004)

253 fb^{-1} :

Select $\rho^+\rho^-$ region:
 $(0.62 < m_{\pi\pi} < 0.92 \text{ GeV}/c^2)$



Select $B^0 \rightarrow 4\pi$ region in
 $(m_{bc}, \Delta E)$:



2-d m_{bc} - ΔE fit:

$$N_{\rho\rho + \text{nonres}} = 207 \pm 29$$

$m_{\pi\pi}$ fit:

$$f_{\text{nonres}} = (6.3 \pm 6.7)\%$$

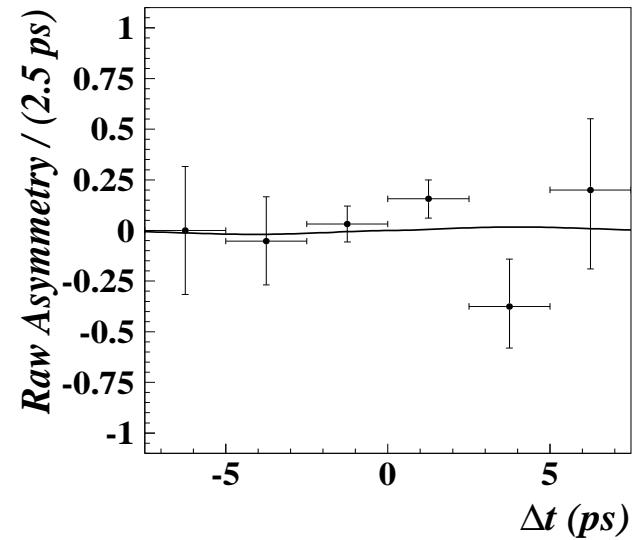
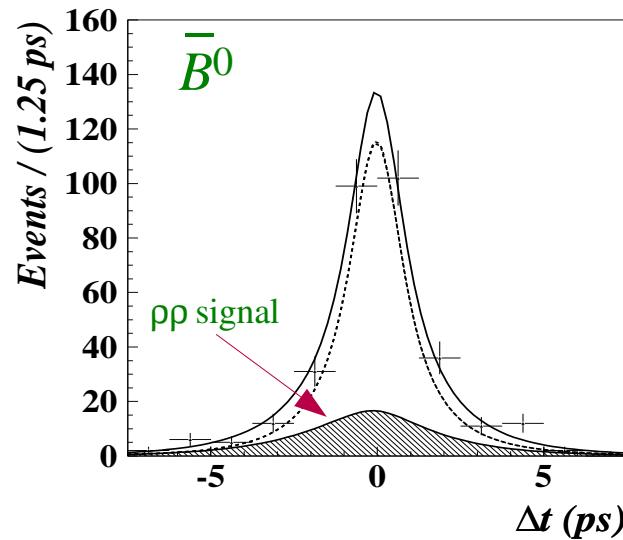
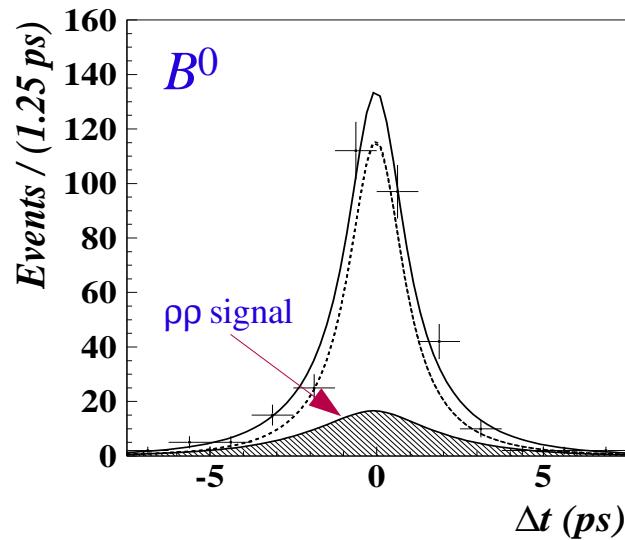
$$\Rightarrow N_{\rho\rho} = 194 \pm 32$$

$$\mathcal{B} = (22.8 \pm 3.8 \pm 2.5) \times 10^{-6}$$

$\cos \theta$ (helicity) fit:

$$f_{\text{longitudinal}} = (94 \pm 4 \pm 3)\%$$

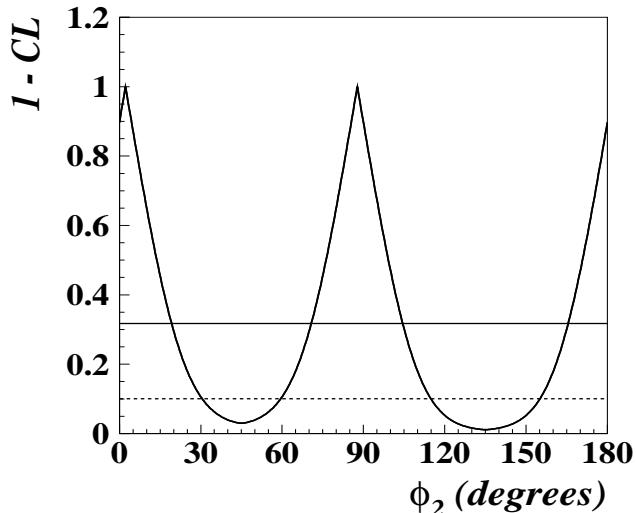
253 fb^{-1} :



$$A_{\rho\rho} = 0.00 \pm 0.30 \pm 0.09$$

$$S_{\rho\rho} = 0.08 \pm 0.41 \pm 0.09$$

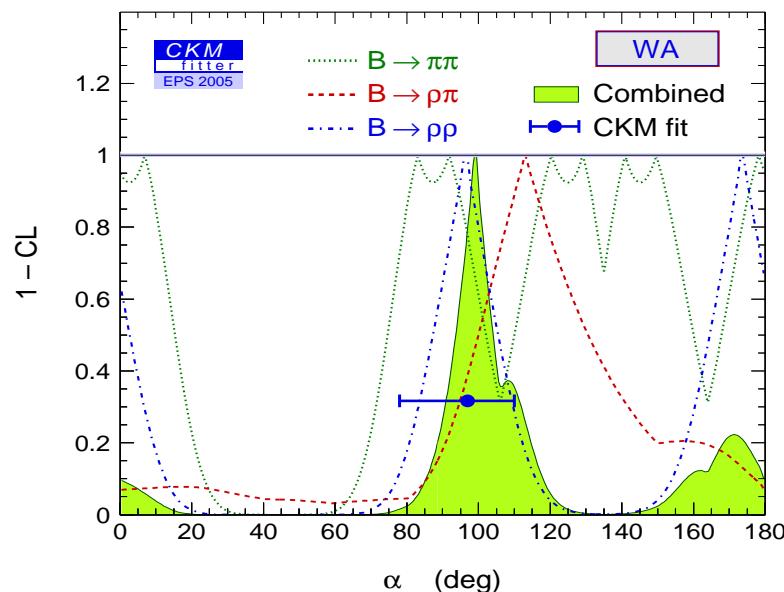
no CPV

253 fb^{-1} :


CL from cumulative χ^2
distribution [$A(\rho^0\rho^0)$ missing]:

$$\phi_2 = (87 \pm 17)^\circ$$

$$59^\circ < \phi_2 < 115^\circ \quad (90\% \text{ CL})$$



CKM fitter Belle +BaBar:

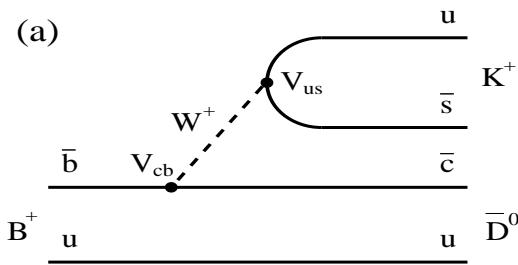
$$\phi_2 = (99^{+13}_{-8})^\circ$$

(<http://ckmfitter.in2p3.fr>)

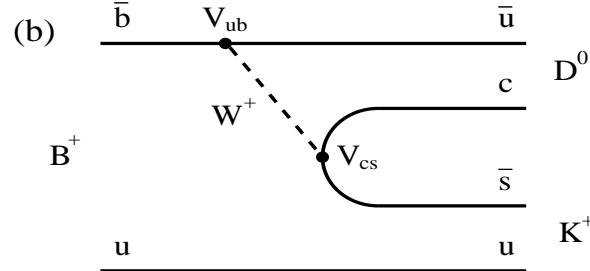
Measurement of ϕ_3

A. Bondar *et al.*, 2002 (unpublished);
 Giri *et al.*, PRD 68, 054018, 2003

$$B^+ \rightarrow \bar{D}^{0(*)} K^+$$



$$B^+ \rightarrow D^{0(*)} K^+$$



if $\bar{D}^0/D^0 \rightarrow K_s \pi^+ \pi^-$, amplitudes interfere

$$m_+ = m(K_s^0, \pi^+)$$

$$m_- = m(K_s^0, \pi^-)$$

$$r = \left| \frac{A(B^+ \rightarrow D^0 K^+)}{A(B^+ \rightarrow \bar{D}^0 K^+)} \right| \sim 0.1 - 0.2$$

$$M_+ = A(m_+^2, m_-^2) + r e^{i(\delta + \phi_3)} A(m_-^2, m_+^2)$$

$$M_- = A(m_-^2, m_+^2) + r e^{i(\delta - \phi_3)} A(m_+^2, m_-^2)$$

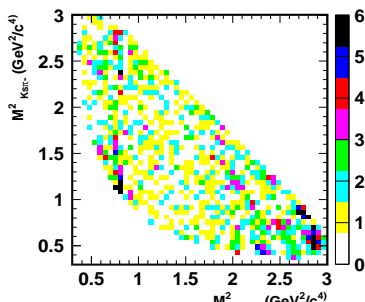
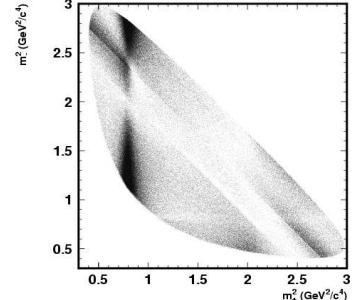
$$\begin{aligned} |M_{\pm}|^2 &= (r^2)_- |A(m_+^2, m_-^2)|^2 + (r^2)_+ |A(m_-^2, m_+^2)|^2 + \\ &\quad 2 |A(m_+^2, m_-^2)| |A(m_-^2, m_+^2)| r \cos(\delta + \theta_{(m_+^2, m_-^2)} \pm \phi_3) \end{aligned}$$

amplitude A determined from $D^0 \rightarrow K_s \pi^+ \pi^-$ Dalitz plot (from continuum)



Measurement of ϕ_3 : $D^0 \rightarrow K_s \pi^+ \pi^-$ decay model

357 fb^{-1} preliminary



$\Delta N(\sigma)$

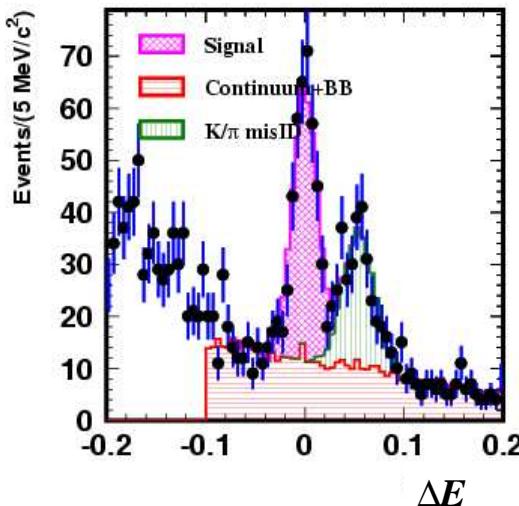
Intermediate state	Amplitude	Phase, $^\circ$	Fit fraction
$K_S \sigma_1$ (M=520±15 MeV, ω =466±31 MeV)	1.43±0.07 1 (fixed) 0.0314±0.0008 0.365±0.006 0.23±0.02	212±4 0 (fixed) 110.8±1.6 201.9±1.9 237±11	9.8% 21.6% 0.4% 4.9% 0.6%
$K_S \rho(770)$			
$K_S \omega$			
$K_S f_0(980)$			
$K_S \sigma_2$ (M=1059±6 MeV, ω =59±10 MeV)	1.32±0.04 1.44±0.10 0.66±0.07 1.644±0.010 0.144±0.004	348±2 82±6 9±8 132.1±0.5 320.3±1.5	1.5% 1.1% 0.4% 61.2% 0.55%
$K_S f_2(1270)$			
$K_S f_0(1370)$			
$K_S(1450)$			
$K^*(892)^+ \pi^-$	0.61±0.06	113±4	0.05%
$K^*(892)^- \pi^+$	0.45±0.04	254±5	0.14%
$K^*(1410)^+ \pi^-$	2.15±0.04	353.6±1.2	7.4%
$K^*(1410)^- \pi^+$	0.47±0.04	88±4	0.43%
$K^*_0(1430)^+ \pi^-$	0.88±0.03	318.7±1.9	2.2%
$K^*_0(1430)^- \pi^+$	0.25±0.02	265±6	0.09%
$K^*_2(1430)^+ \pi^-$	1.39±0.27	103±12	0.36%
$K^*_2(1430)^- \pi^+$	1.2±0.2	118±11	0.11%
$K^*(1680)^+ \pi^-$	3.0±0.3	164±5	9.7%
$K^*(1680)^- \pi^+$			
Nonresonant			



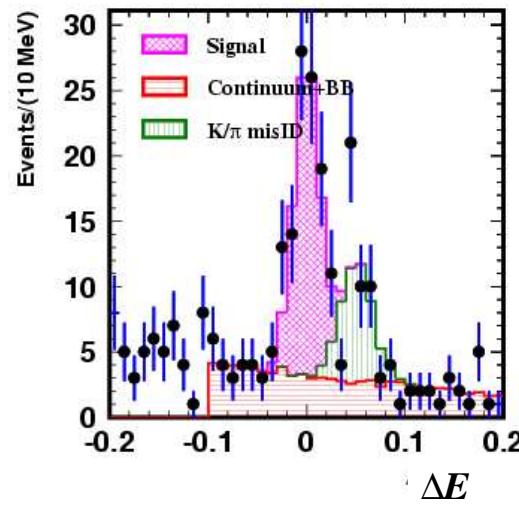
Measurement of ϕ_3

hep-ex/0411049 (253 fb^{-1}) → 357 fb^{-1}

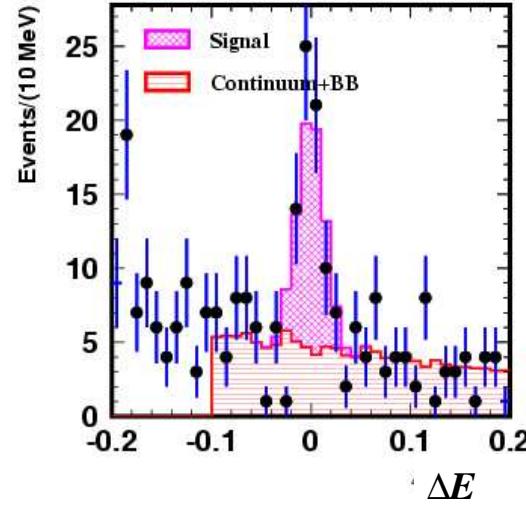
357 fb^{-1} preliminary



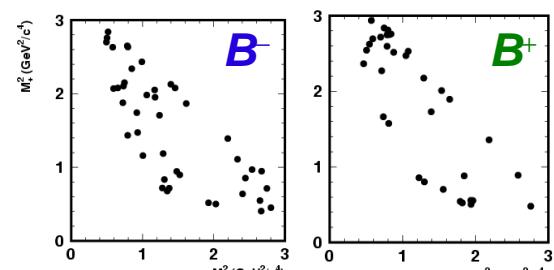
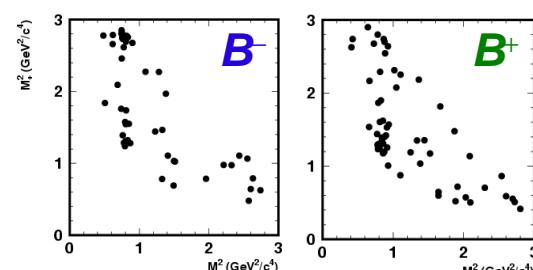
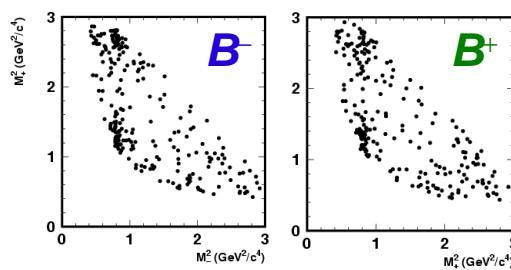
$B^\pm \rightarrow D^0 K^\pm$
 331 ± 17 events
67% pure



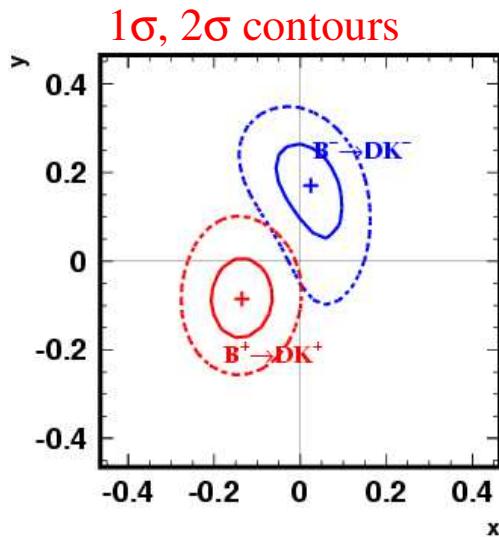
$B^\pm \rightarrow D^{*0} K^\pm$
 81 ± 8 events
77% pure



$B^\pm \rightarrow D^0 K^{*\pm}$
 54 ± 8 events
65% pure



357 fb^{-1} preliminary



$$B^\pm \rightarrow D^0 K^\pm$$

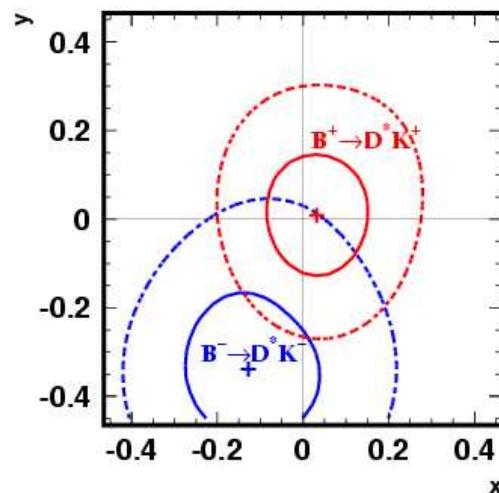
$$x_- = 0.025^{+0.072}_{-0.080}$$

$$x_+ = -0.135^{+0.069}_{-0.070}$$

$$y_- = 0.170^{+0.093}_{-0.117}$$

$$y_+ = -0.085^{+0.090}_{-0.086}$$

Fitted parameters are $x_{\pm} = r \cos(\pm \phi_3 + \delta)$, $y_{\pm} = r \sin(\pm \phi_3 + \delta)$
 (better behaved statistically than r, ϕ_3, δ)



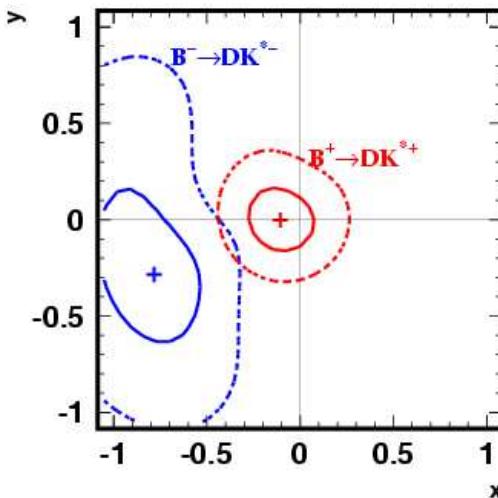
$$B^\pm \rightarrow D^{\ast 0} K^\pm$$

$$x_- = -0.128^{+0.167}_{-0.146}$$

$$x_+ = 0.032^{+0.120}_{-0.116}$$

$$y_- = -0.339^{+0.172}_{-0.158}$$

$$y_+ = 0.008^{+0.137}_{-0.136}$$



$$B^\pm \rightarrow D^0 K^{\ast\pm}$$

$$x_- = -0.784^{+0.249}_{-0.295}$$

$$x_+ = -0.105^{+0.177}_{-0.167}$$

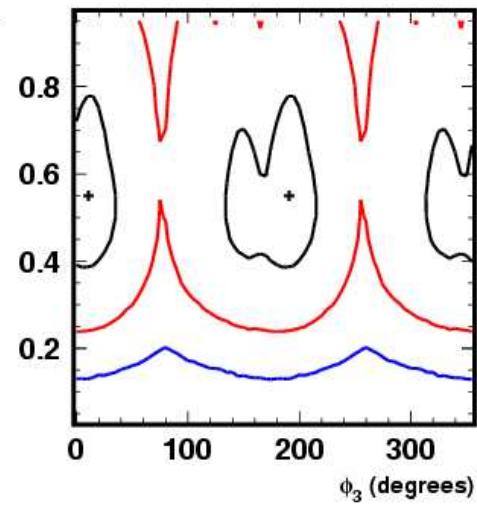
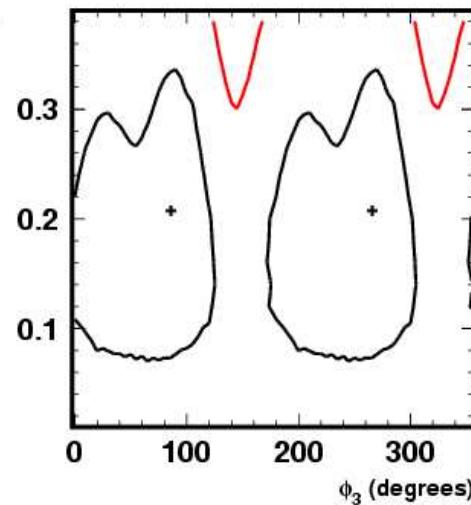
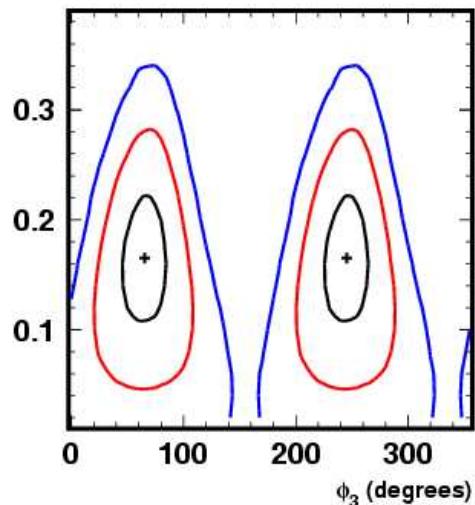
$$y_- = -0.281^{+0.440}_{-0.335}$$

$$y_+ = -0.004^{+0.164}_{-0.156}$$

CPV
sig: 78%

357 fb^{-1} preliminary

Use frequentist toy MC method to obtain CL regions for r , ϕ_3 , δ



$B^\pm \rightarrow D^0 K^\pm$

$\phi_3 = 66^{+19}_{-20}$ (stat)

$r = 0.16 \pm 0.06$

± 0.01 (sys) ± 0.05 (model)

$\delta = 148^{+19}_{-20}$ (stat)

$B^\pm \rightarrow D^{*0} K^\pm$

$\phi_3 = 86^{+37}_{-93}$ (stat)

$r = 0.18 \pm 0.11$

± 0.01 (sys) ± 0.05 (model)

$B^\pm \rightarrow D^0 K^{*\pm}$

$\phi_3 = 11^{+23}_{-57}$ (stat)

$r = 0.56^{+0.22}_{-0.16}$

± 0.04 (sys) ± 0.08 (model)

combined: $\phi_3 = [53^{+15}_{-18} \pm 3 \text{ (sys)} \pm 9 \text{ (model)}]^\circ$
 $8^\circ < \phi_3 < 111^\circ$ (2σ)

Summary I

357 fb⁻¹ final:

$$\sin(2\phi_1): \quad 0.652 \pm 0.039 \pm 0.020 \Rightarrow \boxed{\phi_1 = (20.3^{+1.7}_{-1.6})^\circ}$$

$b \rightarrow qqs$ penguin: consistent with SM, largest difference is 2σ

(before: 2.3σ for average)

253 fb⁻¹ final:

$\sin(2\phi_2)$: we have observed *large CP* violation in $B \rightarrow \pi^+\pi^-$:

$$A_{\pi\pi} = +0.56 \pm 0.12 \text{ (stat)} \pm 0.06 \text{ (syst)}$$

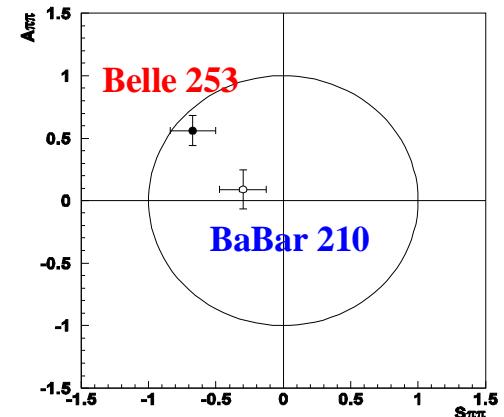
$$S_{\pi\pi} = -0.67 \pm 0.16 \text{ (stat)} \pm 0.06 \text{ (syst)}$$

($A_{\pi\pi}$ indicates direct *CPV* at 4σ significance)

$$|P/T| > 0.17 \text{ (95% CL)} \quad \delta < -4^\circ \text{ (95% CL)}$$

isospin analysis of $B \rightarrow \pi\pi$: $0^\circ < \phi_2 < 19^\circ, 71^\circ < \phi_2 < 180^\circ$ (95% CL)

isospin analysis of $B \rightarrow \rho\rho$: $\boxed{\phi_2 = (87 \pm 17)^\circ} \quad 59^\circ < \phi_2 < 115^\circ$ (90% CL)



357 fb⁻¹ preliminary:

ϕ_3 :

$$(53^{+15}_{-18} \pm 3 \pm 9)^\circ$$

$$8^\circ < \phi_3 < 111^\circ \text{ (95% CL)}$$

significance of *CP* violation = 78%

Summary II

- CKM fitter**
- $\phi_1 = (22 \pm 1.2)^\circ$
- Belle + BaBar:**
- $\phi_2 = (99 \pm 13 - 8)^\circ$
 - $\phi_3 = (63 \pm 15 - 12)^\circ$
- (Belle: 253 fb^{-1})

Does the triangle close?

$$\begin{aligned}\phi_1 + \phi_2 + \phi_3 - 180^\circ \\ = (4^{+20}_{-15})^\circ\end{aligned}$$

Yes! ...and...

outstanding agreement
with other measurements
(ε , V_{ub} , B^0 - \bar{B}^0 mixing)

